A Power Control Game Based on Outage Probabilities for Multicell Wireless Data Networks

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***Abstract*— We present a game-theoretic treatment of distributed power control in CDMA wireless systems using outage probabilities. We prove that the noncooperative power control game considered admits a unique Nash equilibrium (NE) for uniformly strictly convex pricing functions and under some technical assumptions on the SIR threshold levels. We analyze global convergence of continuous-time as well as discrete-time synchronous and asynchronous iterative power update algorithms to the unique NE of the game. Furthermore, a stochastic version of the discrete-time update scheme, which models the uncertainty due to quantization and estimation errors, is shown to converge almost surely to the unique NE point. We further investigate and demonstrate the convergence and robustness properties of these update schemes through simulation studies.**

I. Introduction

The primary objective of uplink power control in code division multiple access (CDMA) wireless networks is to achieve and maintain a satisfactory level of service, which may be described in terms of signal-to-interference ratio (SIR). Since in CDMA systems signals of other users can be modeled as interfering noise signals, there is a tradeoff between the individual objectives of mobiles and the overall system performance. If mobiles have different preferences for the level of service or varying SIR requirements, then the power control problem can be posed as one of resource allocation. Furthermore, under a distributed power control regime the mobiles cannot have detailed information on each other's preferences and actions due to communication constraints inherent to the system. It is, hence, appropriate to address CDMA uplink power control within a noncooperative game theoretic framework, where Nash equilibrium (NE) provides a relevant solution concept. The power control game can also be extended by making use of pricing. A pricing scheme not only enhances the overall system performance by limiting the interference [1], but also results in battery energy preservation.

Several studies exist in the literature that use game theoretic schemes to address the power control problem [1]–[6]. The power control game leads to distributed power control algorithms as a mean to maintain service level under varying channel conditions. An important aspect of a distributed power control scheme is the convergence properties of algorithms, which plays a significant role in performance of the system. The study [7] has presented a standard power control algorithm, and has established its synchronous and asynchronous convergence under some conditions on the interference function. In [8], stochastic power control schemes have been investigated, and the converge of stochastic algorithms in terms of mean-squared error has been proven. Another study [9] has shown the convergence of a coupled power control scheme based on minimum outage probability and multiuser detection by making use of standard interference functions of [7]. In [5], two update algorithms, namely, parallel update and random update have been shown to be globally stable under specific conditions. Finally, in [6] the global convergence of the dynamics of the power control game to a superset of Nash equilibria has been established for any handoff scheme satisfying a mild condition on average dwell time.

In this paper, we consider a power control game similar to the one in [6], which incorporates a pricing mechanism limiting the overall interference and preserving battery energy of mobiles. We capture the preferences of mobiles using a utility function, which is defined as the logarithm of the probability that the SIR level of the mobile is greater than a predefined individual threshold level. This utility function can also be described in terms of outage probabilities [9]. The noncooperative power control game obtained admits a unique Nash equilibrium under uniformly strictly convex pricing functions and some technical assumptions on the SIR threshold levels. Furthermore, we investigate global convergence of a continuous-time as well as discrete-time synchronous and asynchronous iterative power update algorithms to the unique NE of the game. A stochastic version of the discrete-time update scheme, which models the uncertainty due to quantization and estimation errors, is shown to converge to the NE almost surely under some conditions. The convergence and robustness properties of these schemes are demonstrated through MATLAB simulations.

The next section describes the model adopted and the cost function. Section III discusses the existence and uniqueness of the Nash equilibrium. We present in Section IV system dynamics and stability analysis of a continuous-time update scheme. In Section V, convergence properties of both deterministic and stochastic discrete-time update algorithms are investigated. Section VI contains results of the simulation studies. The paper concludes with a summary of the completed work and directions for future research in Section VII.

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II. THE MODEL

We consider a multicell CDMA wireless network model similar to the ones described in [3], [9]. The system consists of a set $\mathcal{L} := \{1, \ldots, \bar{L}\}$ of cells, with the set of users in cell l being $\mathcal{M}_l := \{1, \ldots, M_l\}$, $l \in \mathcal{L}$, and the set of all users is defined as $\mathcal{M} := \bigcup_l \mathcal{M}_l$. The number of users in each cell is limited through an admission control scheme. We associate a single base station (BS) with each cell in the system, and define $h_{il} f_{il} p_i$ as the instantaneous received power level from user i at the l^{th} BS. To simplify the analysis, we let a mobile connect to a single BS at any given time. The quantities h_{il} $(0 < h_{il} < 1)$ and f_{il} $(f_{il} > 0)$ represent the *slow-varying* channel gain (excluding any fading) and fast time-scale Rayleigh fading between the i^{th} mobile and the l^{th} BS, respectively. We assume that the factors affecting h_{il} do not change significantly over the time scale of this analysis, and the terms f_{il} are unit mean independent exponentially distributed random variables.

Let $\mathcal{M}_{l,eff}$ denote the set of users in the neighborhood of cell l who have a nonnegligible effect on each other's SIR levels through in-cell and intra-cell interference. It immediately follows that, $\mathcal{M}_l \subset M_{l,eff} \subset \mathcal{M}$. Without loss of any generality, we define the set $\mathcal{M}_{l,eff}$ in this study as

$$\mathcal{M}_{l,eff} := \mathcal{M}_l \cup \left(\cup_{k \in Neighbor(l)} \mathcal{M}_k \right),$$

where $Neighbor(l)$ is defined as the set of first-tier neighbors of the cell l. Furthermore, the contribution of mobiles in other cells to the interference level of cell l is modeled as a fixed background noise, of variance σ_l^2.

The i^{th} mobile transmits with a nonnegative uplink power level of $p_i \leq p_{i,max}$, where $p_{i,max}$ is an upper-bound imposed by physical limitations of the mobile. Thus, in accordance with the interference model considered the SIR obtained by mobile i at the base station l is given by

$$\gamma_{il} := \frac{L h_{il} f_{il} p_i}{\sum_{j \in \mathcal{M}_{l,eff},\, j \neq i} h_{jl} f_{jl} p_j + \sigma_l^2}. \quad (1)$$

Here, $L := W/R > 1$ is the spreading gain of the CDMA system, where W is the chip rate and R is the data rate of the user. The outage probability of user i, denoted O_{il}, is defined as the proportion of time that some SIR threshold, $\bar{\gamma}_{il}$, is not met for sufficient reception at the l^{th} BS receiver [9]. By a careful choice of $\bar{\gamma}_{il}$, a quality of service level can be established for each user. The outage probability, $O_{il} = Pr(\gamma_i \leq \bar{\gamma}_{il})$, of the i^{th} mobile at the l^{th} BS is defined as

$$O_{il} = Pr\left(h_{il} f_{il} p_i \leq \bar{\gamma}_{il} \left[\sum_{j \in \mathcal{M}_{l,eff},\, j \neq i} h_{jl} f_{jl} p_j + \sigma_l^2 \right] \right), \quad (2)$$

where $Pr(\gamma_i \leq \bar{\gamma}_{il})$ denotes the probability of $\gamma_i \leq \bar{\gamma}_{il}$.

In the Rayleigh/Rayleigh fading environment described, the mean power level of mobile i received at the l^{th} BS is defined as $x_{il} := h_{il} p_i$. Let the received power level vector of cell l be $\mathbf{x}_l := [(x_{j_l l})], j \in \mathcal{M}_{l,eff}$. Then, the system wide vector $\mathbf{x} := [\mathbf{x}_1, \ldots, \mathbf{x}_L]$ has the cardinality $Mx := \sum_{l \in \mathcal{L}} M_{l,eff}$, where $M_{l,eff}$ is the number of elements of the set $\mathcal{M}_{l,eff}$. In order to simplify the notation we will drop the index of the BS (e.g. $x_i := x_{il}$) in cases where when it is obvious from the context that mobile i is connected to the l^{th} BS. As a further simplification, we let the threshold SIR for the i^{th} mobile be defined as $\bar{\gamma}_i := \bar{\gamma}_{il} = \bar{\gamma}_{ik}$ $\forall l, k \in \mathcal{L}$. We note that the outage probability in (2) can be expressed in analytical form which we present here without derivation. The derivation of it can be found in [10], and in [11] for a simplified version of the expression. The outage probability of the i^{th} mobile is then given by

$$O_i(\mathbf{x}, \bar{\gamma}_i) = 1 - \exp\left(\frac{-\sigma^2 \bar{\gamma}_i}{x_i} \right) \prod_{j \neq i} \frac{1}{1 + \dfrac{\bar{\gamma}_i x_{jl}}{x_i}}, \quad (3)$$

where $x_{jl} = \frac{h_{jl}}{h_j} x_j$ is the received power level of the j^{th} mobile at the l^{th} BS, and h_j (x_j) is the channel gain (received power level) of it at its own BS. We also note that, for the rest of this paper, the term $j \neq i$ implicitly denotes $j \in \mathcal{M}_{l,eff}$, l being the BS to which mobile i is connected.

The i^{th} user's cost function is defined as the difference between the utility function of the user and its pricing function, $J_i = P_i - U_i$, similar to the one in [5]. The utility function, $U_i(Pr_i(\gamma_i \geq \bar{\gamma}_i)$, is a logarithmic function of the probability that the SIR of the i^{th} user is larger than the predefined threshold, $\bar{\gamma}_i$, and quantifies approximately the demand or *willingness to pay* of the user for a certain level of service. Notice that, $Pr_i(\gamma_i \geq \bar{\gamma}_i)$ is equal to $1 - O_i$, where O_i is the outage probability in (3). Hence, the utility function for the user i is defined by

$$U_i(\mathbf{x}) := u_i \log(Pr_i(\gamma_i(\mathbf{x}) \geq \bar{\gamma}_i) = u_i \log(1 - O_i(\mathbf{x}, \bar{\gamma}_i)), \quad (4)$$

where u_i is a user-specific utility parameter, and

$$\begin{aligned} Pr_i(\gamma_i(\mathbf{x}) \geq \bar{\gamma}_i) = &\ 1 - O_i(\mathbf{x}, \bar{\gamma}_i) \\ = &\ \exp\left(\frac{-\sigma_l^2 \bar{\gamma}_i}{x_i} \right) \prod_{j \neq i} \frac{1}{1 + \frac{\bar{\gamma}_i x_{jl}}{x_i}}. \end{aligned}$$

One can show that U_i is increasing in x_i, its derivative is decreasing in x_i, and $\frac{\partial^2 U_i(\mathbf{x})}{\partial x_i \partial x_{jl}} > 0$ for $j \neq i$.

The pricing function, $P_i(p_i)$, on the other hand, is imposed by the system to limit the interference created by the mobile, and hence to improve the system performance [3]. At the same time, it can also be interpreted as a cost on the battery usage of the user. As a result, the cost function of the i^{th} user connected to a specific BS is given by

$$J_i(\mathbf{x}) = P_i(x_i) - u_i \log(Pr_i(\gamma_i(\mathbf{x}) \geq \bar{\gamma}_i)), \quad (5)$$

where we have used x_i, instead of p_i, as the argument of P_i, by a possible redefinition of the latter.

III. Existence and Uniqueness of Nash Equilibrium

Let us define x_{min} and x_{max} as lower and upper bounds on x_{il} $\forall i, l$, i.e. $x_{min} < x_{il} < x_{max} \forall i, l$. If the mean received power level of a mobile at the BS is less than x_{min}, then its effect is negligible and modeled as part of the background noise. The upper-bound x_{max} is further bounded above by p_{max} with a possible equality in the case of no channel attenuation. We also define $\bar{\gamma}_{min}$ (u_{min}) and $\bar{\gamma}_{max}$ (u_{max}) in such a way that $\bar{\gamma}_{min} < \bar{\gamma}_i < \bar{\gamma}_{max}$ $(u_{min} < u_i < u_{max})$ $\forall i$. We now make the following assumptions on the cost function:

A1. $P_i(x_i)$ is twice continuously differentiable, non-decreasing and uniformly strictly convex in x_i, i.e.

$$dP_i(x_i)/dx_i \geq 0,\ d^2P_i(x_i)/dx_i^2 \geq v > 0,\ \forall x_i,$$

for some $v > 0$.

A2. Given the set of parameters $\{M_{l,eff},\ \bar{\gamma}_{min},\ \bar{\gamma}_{max},\ x_{min},\ x_{max}\}$, v satisfies the following inequality:

$$v(\bar{\gamma}_{min}+1)\frac{x_{min}^2}{u_{max}} + (M_{l,eff}-1)\bar{\gamma}_{min}\frac{u_{min}}{u_{max}}\frac{x_{min}^3}{x_{max}^3} > 1$$

A3. The i^{th} user's cost function has the following properties at $x_i = x_{min}$ $(x_i = x_{max})$: $\partial J_i(\mathbf{x} : x_i = x_{min})/\partial x_i < 0\ \forall \mathbf{x}$ $(\partial J_i(\mathbf{x} : x_i = x_{max})/\partial x_i > 0\ \forall \mathbf{x})$, respectively.

The Nash equilibrium (NE) in a cell is defined as a set of power levels, p^* (and corresponding set of costs J^*), with the property that no user in the cell can benefit by modifying its power level while the other players keep theirs fixed. Mathematically speaking, $\mathbf{x}^*$ is in NE, when x_i^* of any i-th user is the solution to the following optimization problem given the equilibrium power levels of other mobiles (in the set $\mathcal{M}_{l,eff}$), $\mathbf{x}_{-i}^*$:

$$\min_{x_{min} \leq x_i \leq x_{max}} J_i(x_i, \mathbf{x}_{-i}^*). \tag{6}$$

Note that given the channel gains the NE point $\mathbf{x}^*$ is equivalent to p^*.

Thanks to assumption *A1*, the cost function J_i is strictly convex and belongs to a fairly large subclass of convex functions. Hence, there exists a unique solution to the i^{th} user's minimization problem, which is that of minimization of J_i, given the system parameters and the power levels of all other users. The technical assumption *A2* is needed for the proof of existence of a unique NE. Notice that, $\mathbf{x}_{min}$ is bounded below by definition. Hence, *A2* is easily satisfied for a large number of users M or high SIR thresholds $\bar{\gamma}_{min}$ even if v is small. Assumption *A3*, on the other hand, ensures that any equilibrium solution is an *inner* one, i.e., boundary solutions $x_i^* = x_{min}$ $(x_i^* = x_{max})$ $\forall i$ cannot be equilibrium points.

Theorem III.1. *Under A1-A3, the multicell power control game defined admits a unique inner Nash equilibrium solution.*

IV. System Dynamics and Stability Analysis

We consider a dynamic model of the power control game similar to the one of [6] where each mobile uses a gradient algorithm to solve its own optimization problem (6). Hence, the following analysis is similar to the one in [6]. The power update algorithm of the i^{th} mobile is:

$$\dot{p}_i = \frac{dp_i}{dt} = -\frac{\partial J_i}{\partial p_i},$$

for all $i \in \mathcal{M}$. This can also be described in terms of the received power level, x_i, at the l^{th} BS:

$$\dot{x}_i = h_i^2\left(\frac{\partial U_i(\mathbf{x})}{\partial x_i} - \frac{dP_i(x_i)}{dx_i}\right) := \phi_i(\mathbf{x}),\ \forall i. \tag{7}$$

By taking the second derivative of x_i with respect to time, we obtain

$$\ddot{x}_i = h_i^2\left(-a_i - \frac{d^2P_i(x_i)}{dx_i^2}\right)\dot{x}_i + h_i^2\sum_{j\neq i} b_{i,j}\dot{x}_{jl} := \dot{\phi}_i(\mathbf{x}), \tag{8}$$

where a_i and $b_{i,j}$ are defined as

$$a_i := -\frac{\partial^2 U_i(\mathbf{x})}{\partial x_i^2} = u_i\frac{2\sigma_l^2 + \bar{\gamma}_i}{x_i^3} + u_i\sum_{j\neq i}\frac{1 + \dfrac{2x_i}{\bar{\gamma}_i x_{jl}}}{\left(x_i + \dfrac{x_i^2}{\bar{\gamma}_i x_{jl}}\right)^2},$$

and

$$b_{i,j} := \frac{\partial^2 U_i(\mathbf{x})}{\partial x_i\,\partial x_{jl}} = u_i\frac{\bar{\gamma}_i}{(x_i + \bar{\gamma}_i x_{jl})^2}.$$

Notice that, both a_i and $b_{i,j}$ are positive.

We establish the stability of the power update scheme (7) under some sufficiency conditions. The set of feasible received power levels is invariant by assumption *A3*, which immediately follows from a boundary analysis. When $x_i = x_{min}$ for some $i \in \mathcal{M}$, we have $\dot{x}_i > 0$ under *A3*. Hence, the system trajectory moves toward inside of X. Likewise, in the case of $x_i = x_{max}$ for some $i \in \mathcal{M}$, $\dot{x}_i < 0$, and hence, the trajectory remains inside the set X. Let us define the candidate Lyapunov function $V : \mathbb{R}^{Mx} \to \mathbb{R}$ as $V(\mathbf{x}) := \sum_{i\in\mathcal{M}}(1/h_i^2)\phi_i^2(\mathbf{x})$, which is in fact restricted to the domain X. Note that because of the uniqueness of the NE, $\mathbf{x}^*$, $\phi_i(\mathbf{x}) = 0\ \forall i$ if and only if $\mathbf{x} = \mathbf{x}^*$. Hence, V is positive for all $\mathbf{x}$ except for $\mathbf{x} = \mathbf{x}^*$. Let us now change assumption *A2* as follows:

A2'. Assume that the following inequality holds:

$$v(\bar{\gamma}_{min}+1)\tfrac{x_{min}^2}{u_{max}} + (M_{l,eff}-1)\bar{\gamma}_{min}\frac{u_{min}}{u_{max}}\frac{x_{min}^3}{x_{max}^3} > M_{eff} - 1\ \ \forall l.$$

Remark IV.1. *A2'* can be satisfied by choosing $\bar{\gamma}_{min}$ and/or v sufficiently large.

Then, under *A2'*, we have $\dot{V}(\mathbf{x}) < 0$, uniformly in the x_i's on the trajectory of (7). Thus, V is indeed a Lyapunov function, and it readily follows that $\phi_i(\mathbf{x}(t)) = \dot{x}_i(t) \to 0,\ \ \forall i$. This in turn implies that $x_i(t)$'s converge to the unique Nash equilibrium. Hence, the unique NE point (Theorem III.1) is globally asymptotically stable on the invariant set X with respect to the update scheme (7) under the assumptions *A1, A2$'$, A3* by Lyapunov's stability theorem (see Theorem 3.1 in [12]).

V. Iterative Power Control Algorithms

We investigate stability properties of synchronous and asynchronous iterative power control schemes as they are of practical importance. We first analyze gradient based synchronous and asynchronous update algorithms of the power control game in Section III. Consequently, we study convergence of stochastic iterations to the unique NE solution by taking communication constraints and estimation errors into account.

A. Synchronous and Asynchronous Update Schemes

Consider a discrete-time counterpart of the update scheme in (7) in a system with M mobiles where each mobile uses a gradient algorithm to solve its optimization problem (6):

$$p_i(n+1) = p_i(n) - \lambda_i \frac{\partial J_i}{\partial p_i} \;\; \forall i \in \mathcal{M} \;,$$

where $n = 1, 2, \ldots$, denotes the update instances and λ_i is the user-specific step size constant. For notational convenience this can also be defined as a mapping from the received power levels at the BS to the updated power levels, $\mathbf{x}(n+1) = T(\mathbf{x}(n))$, i.e.

$$x_i(n+1) = T_i(\mathbf{x}(n)) := x_i(n) - \lambda \frac{\partial J_i}{\partial x_i} \;\; \forall i \in \mathcal{M} \;. \quad (9)$$

In the case of synchronous update algorithm each mobile updates its power level at the same time instance. We study sufficient conditions for convergence of the system to the unique NE, $\mathbf{x}^*$, under the synchronous update. This analysis follows lines similar to those in the proof of Proposition 1.10 of [13, p. 193]. Let $\mathbf{x} \in X = \{\mathbf{x} \in \mathbb{R}^{Mx} : x_{min} \leq x_{il} \leq x_{max} \; \forall i, l\}$ and define a function $g_i(\tau) : [0,1] \to \mathbb{R}$ for the i^{th} mobile by

$$g_i(\tau) = \tau x_i + (1-\tau)x_i^* + \lambda \phi_i(\tau \mathbf{x} + (1-\tau)\mathbf{x}^*),$$

where ϕ_i is defined in (7). We then have

$$|T_i(\mathbf{x}) - T_i(\mathbf{x}^*)| = |g_i(1) - g_i(0)| = \left| \int_0^1 \frac{dg_i(\tau)}{d\tau} d\tau \right|$$

$$\leq \int_0^1 \left| \frac{dg_i(\tau)}{d\tau} \right| d\tau \leq \max_{\tau \in [0,1]} \left| \frac{dg_i(\tau)}{d\tau} \right|,$$

where $\mathbf{x}^*$, the NE, is the fixed point of the mapping T. We bound $\left| \frac{dg_i(\tau)}{d\tau} \right|$ above by

$$\left| \frac{dg_i(\tau)}{d\tau} \right| \leq \left| x_i - x_i^* - \lambda \sum_{j \in \mathcal{M}_{l,eff}} \frac{\partial \phi_i}{\partial x_j} \cdot (x_j - x_j^*) \right|$$

$$\leq \left| 1 - \lambda \frac{\partial \phi_i}{\partial x_i} \right| |x_i - x_i^*| + \sum_{j \neq i} \lambda \frac{\partial \phi_i}{\partial x_{jl}} \left| x_{jl} - x_{jl}^* \right| .$$

Imposing the condition $\lambda \partial \phi_i / \partial x_i < 1$, we have

$$\left| \frac{dg_i(\tau)}{d\tau} \right| \leq \left(1 - \lambda \left[\frac{\partial \phi_i}{\partial x_i} - \sum_{j \neq i} \frac{\partial \phi_i}{\partial x_{jl}} \right] \right) \| \mathbf{x} - \mathbf{x}^* \| ,$$

where $\|\mathbf{x}\| := \max_i |x_i|$ is the maximum norm. Define

$$K_i := \max_{\mathbf{x} \in X} \frac{\partial \phi_i(\mathbf{x})}{\partial x_i} \text{ and } \rho_i := 1 - \lambda \left(\frac{\partial \phi_i}{\partial x_i} - \sum_{j \neq i} \frac{\partial \phi_i}{\partial x_{jl}} \right),$$

which leads to $|T_i(\mathbf{x}) - x_i^*| \leq \rho_i \|\mathbf{x} - \mathbf{x}^*\|$ for each i. Let $\rho := \max_i \rho_i$ and $K := \max_i K_i$. We obtain then $\|T(\mathbf{x}) - \mathbf{x}^*\| \leq \rho \|\mathbf{x} - \mathbf{x}^*\|$, if $\lambda K < 1$. An upper bound on K in terms of system and cost parameters is

$$\bar{K} := \max_i \frac{d^2 P_i(x_{max})}{dx_i^2} + \frac{2(M_{eff}-1)\bar{\gamma}_{max} x_{max}}{(\bar{\gamma}_{min}+1)x_{min}^3} + \frac{2\sigma^2 \bar{\gamma}_{max}}{x_{min}^3}.$$

Imposing the condition $\rho < 1$, it readily follows that for arbitrary $\mathbf{x} \in X$, $T^n(\mathbf{x}) \to \mathbf{x}^*$ as $n \to \infty$, since $\|T^n(\mathbf{x}) - \mathbf{x}^*\| \leq \rho^n \|\mathbf{x} - \mathbf{x}^*\|$. Furthermore, the condition $\rho < 1$ is satisfied if

$$\sum_{j \neq i} \frac{\bar{\gamma}_i^2 x_{jl}^2 + 2\bar{\gamma}_i x_i x_{jl}}{x_i^2 (x_i + \bar{\gamma}_i x_{jl})^2} - \frac{\bar{\gamma}_i}{(x_i + \bar{\gamma}_i x_{jl})^2} > 0 \; \forall i.$$

Let $x_{max} = \alpha x_{min}$ for some $\alpha > 0$. Then, a sufficient condition for $\rho < 1$ is $\alpha < 1 + \sqrt{1 + \bar{\gamma}_{min}}$, which follows from a straightforward algebraic derivation. This leads to the following theorem:

Theorem V.1. *Let $x_{max} = \alpha x_{min}$ for some $\alpha > 0$ and $X := \{\mathbf{x} \in \mathbb{R}^M x : x_{min} \leq x_{il} \leq x_{max} \; \forall i, l\}$. The synchronous power update algorithm*

$$p_i(n+1) = p_i(n) - \lambda_i \frac{\partial J_i}{\partial p_i} \;\; \forall i \in \mathcal{M}$$

converges to the unique NE point of the power control game, $p^* := [x_1^*/h_1, \ldots, x_M^*/h_M]$, on the set X if $\lambda \bar{K} < 1$ and $\alpha < 1 + \sqrt{1 + \bar{\gamma}_{min}}$.

A natural generalization of the synchronous update is the asynchronous update scheme, which is in fact more realistic since it is difficult for the mobiles to synchronize their exact power update instances in a practical implementation. In this particular case, however, the convergence analysis above also applies to the asynchronous update algorithm. Define a sequence of nonempty, convex, and compact sets

$$X(k) := [x_1^* - \delta(k), x_1^* - \delta(k)] \times [x_2^* - \delta(k), x_2^* - \delta(k)]$$

$$\times \ldots [x_M^* - \delta(k), x_M^* - \delta(k)],$$

where $\delta(k) := \|\mathbf{x}(k) - \mathbf{x}^*\|$. Since by Theorem V.1, $\delta(k+1) < \delta(k)$, we have

$$\ldots \subset X(k+1) \subset X(k) \subset \ldots X.$$

We next consider the two well known sufficient conditions for asynchronous convergence of a nonlinear iterative mapping $\mathbf{x}(n+1) = T(\mathbf{x})$ [13, p. 431], namely synchronous convergence condition and box condition. Both are satisfied in our case by definition of $X(k)$ and Theorem V.1. Therefore, it immediately follows from asynchronous convergence theorem [13, p. 431] that the asynchronous counterpart of the power update algorithm in (9) converges to the unique NE point of the power control game.

B. A Stochastic Update Scheme

In a real life implementation of the power control scheme, communication constraints, approximations, estimation and quantization errors are not negligible, and have to be taken into account in the convergence analysis. Hence, a mobile does not have access to the exact values of the system parameters such as its own channel gain or the feedback terms provided by the BS. These uncertainties can be captured by defining a stochastic update algorithm for analysis purposes. For each $i \in \mathcal{M}$, let $\xi_i(n)$ $n = 1, 2, \ldots$ be a sequence of independent identically distributed (iid) random variables defined on the common support set $[1-\varepsilon, 1+\varepsilon]$, where $0 < \varepsilon < 1$. We further assume that for each $i \in \mathcal{M}$ the sequence ξ_i is independent of the past of ξ_j, $j \neq i$, that is $Pr(\xi_i(n+1)|\xi_j(s), s \leq n, j \in \mathcal{M}, j \neq i) = Pr(\xi_i(n+1)$. Using these random sequences, we model the aggregate uncertainty in the term $\partial J_i/\partial p_i$ of (9) due to quantization, estimation, and multiplicatively approximation errors. Thus, the stochastic counterpart of the synchronous update algorithm is given by

$$p_i(n+1) = p_i(n) - \lambda_i \xi_i(n) \frac{\partial J_i}{\partial p_i} \quad \forall i \in \mathcal{M}, \qquad (10)$$

which can also be described in terms of received power levels at the base station as

$$\begin{aligned} x_i(n+1) \quad &= x_i(n) - \lambda \xi_i(n) \frac{\partial J_i}{\partial x_i} \\ &=: T_i(\mathbf{x}(n); \xi_i(n)) \;\; \forall i \in \mathcal{M}. \end{aligned} \qquad (11)$$

Following steps similar to those in the previous subsection for the convergence analysis, we arrive at the following theorem (details can be found in the longer version of the paper, available from the authors).

Theorem V.2. *Let $\mathbf{x}_i(n)$ $(\xi_i(n))$ be random (random iid) sequences for all i, where ξ_i is associated with the probability density function $f_{\xi_i}(\xi_i)$ defined on the support set $[1-\varepsilon, 1+\varepsilon]$, $0 < \varepsilon < 1$, and the random vector $\mathbf{x}$ takes its values on the set $X := \{\mathbf{x} \in \mathbb{R}^{Mx} : x_{min} \leq x_{il} \leq x_{max} \; \forall i, l\}$. Furthermore, let $\alpha > 0$ be defined as $\alpha := x_{max}/x_{min}$. The stochastic power update algorithm*

$$p_i(n+1) = p_i(n) - \lambda \xi_i(n) \frac{\partial J_i}{\partial p_i} \quad \forall i \in \mathcal{M},$$

converges almost surely to the unique NE point of the power control game, p^*, if

$$\alpha < \frac{1}{2}\sqrt{\bar{\gamma}_{min}} + \frac{1}{4}$$

and $\lambda(1+\varepsilon)K < 1$ hold, where

$$K = \max_i \frac{d^2 P_i(x_{max})}{dx_i^2} + \frac{2(M_{eff}-1)\bar{\gamma}_{max} x_{max}}{(\bar{\gamma}_{min}+1)x_{min}^3} + \frac{2\sigma^2 \bar{\gamma}_{max}}{x_{min}^3}.$$

VI. Simulations

The power control game based on outage probabilities is simulated in MATLAB for a wireless network consisting of six arbitrarily placed base stations and 20 mobiles. The channel gain of the i^{th} mobile is determined by the log-normal shadowing path loss model given by $h_i = (0.1/d_i)^{2.5} + Y_\sigma^{-1}$, where d_i denotes the distance to the BS and $\log(Y_\sigma)$ is a zero-mean Gaussian random variable with a standard deviation of $\sigma = 0.1$. The loss exponent is chosen as 2.5 which corresponds to a low density urban environment. Each mobile connects to a single BS, which happens to be in the closest geographical location. Hence, the cells in the network are irregularly shaped polygons. The system parameters are chosen as $L = 128$ and $\sigma_l^2 = 1 \; \forall l$.

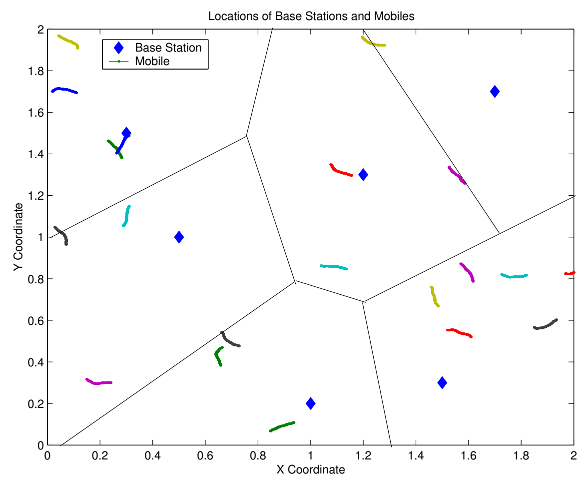

Fig. 1. Locations of base stations and the paths of mobiles.

The mobiles are initially distributed randomly in the network, and their movement is modeled after a two-dimensional random walk with a speed of 0.0001 units per update. Assuming an update frequency of $100Hz$ and geographical unit size of $1000m$, they move with a speed of $10m/s$ or $36km/h$. Figure 1 depicts the locations of the BSs and the path of all mobiles.

The cost function for the i^{th} user (mobile) is chosen as $J_i(\mathbf{x}) = 0.5\alpha_i x_i^2 - u_i \log(Pr_i(\gamma_i(\mathbf{x}) \geq \bar{\gamma}_i))$, where pricing and utility parameters are $u_i = \alpha_i = 1$ and $\bar{\gamma}_i = 100$, which are chosen to be the same for all users for comparison purposes. We first simulate a discrete update scheme with perfect information for the i^{th} user:

$$\begin{aligned} p_i(n+1) \quad &= p_i(n) + \lambda \frac{\sigma_l^2 \bar{\gamma}_i}{h_{il}^2 p_i^2(n)} \\ &+ \frac{\lambda}{h_{il} p_i(n)} \sum_{j \neq i} \frac{1}{1 + \frac{h_{il} p_i(n)}{h_{jl} p_j(n) \bar{\gamma}_i}} - \lambda \alpha h_i p_i(n), \end{aligned} \qquad (12)$$

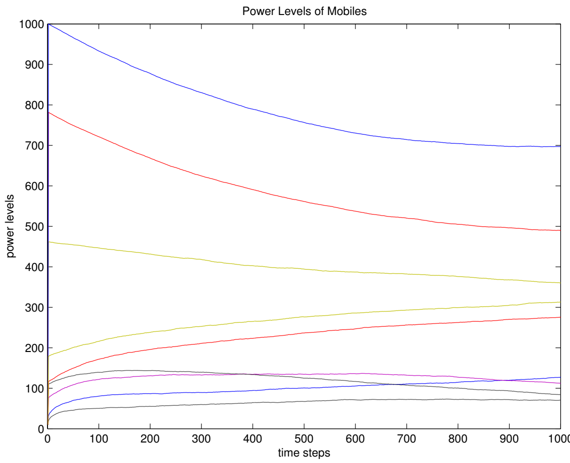

Fig. 2. Power levels of selected mobiles with respect to time.

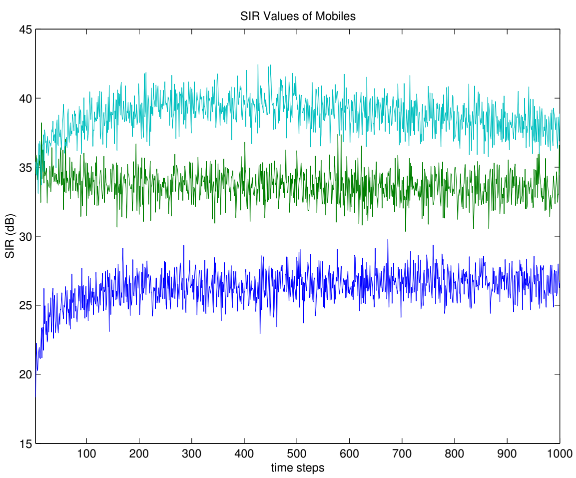

Fig. 3. SIR values of selected mobiles (in dB) with respect to time.

where $\lambda = 0.1\ \forall i$ and n denotes the time. The power levels and SIR values of a randomly selected subset of mobiles for the duration of the simulation are shown in Figures 2 and 3, respectively. The power levels converge to the equilibrium points, which shift due to handoffs in the system. The level changes in SIR values in Figure 3 are also due to the handoffs and variations in interference levels between different cells.

We next consider a more realistic information feedback scheme, where we take into account the distortion in feedback information due to quantization and other effects. Multiplying the parameter λ in the update algorithm (12) with ξ, which is a random variable uniformly distributed on $[0.9, 1.1]$, we rerun the simulation with this imperfect feedback algorithm. In accordance with Theorems V.1 and V.2, convergence characteristics of the system are not significantly affected.

VII. Conclusions

In this paper, we have considered a power control game similar to the one in [6] with a utility function, which is defined as the logarithm of the probability that the SIR level of the mobile is greater than a predefined individual threshold level. Hence, we have established a relationship between the preferences of the mobiles and outage probabilities. We have proven that the noncooperative power control game admits a unique Nash equilibrium for uniformly strictly convex pricing functions and under some technical assumptions on the SIR threshold levels. Furthermore, we have established the global convergence of continuous-time as well as discrete-time synchronous and asynchronous iterative power update algorithms to the unique NE of the game under some conditions. Likewise, a stochastic version of the discrete-time update scheme, which models the uncertainty due to quantization and estimation errors, has been shown to converge to the unique NE point almost surely.

Finally, through extensive simulation studies we have demonstrated the convergence and robustness properties of power update schemes developed. Some of the possible future extensions of this study include the simulation of asynchronous update schemes as well as analysis and simulation of various handoffs algorithms.

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