

Resource Allocation Game for Wireless Networks with Queue Stability Constraints

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Abstract—We study the interaction among users of contention-based wireless networks, where the performance of the network is highly correlated with user transmission probabilities. Considering the underlying user incentives, we make use of the conceptual framework of noncooperative game theory to obtain a distributed control mechanism to limit the contention among wireless nodes by taking into account queue stability. We present a comprehensive analysis of the game including existence and uniqueness of Nash equilibrium point and convergence dynamics. Utilizing linear pricing enables us to move the equilibrium point of the game to a desirable region. We obtain conditions on linear prices necessary to achieve stability of user queues in the asymmetric and symmetric cases.

I. INTRODUCTION

Contention-based medium access control plays a significant role in the successful deployment of modern wireless networks, where users are expected to manage their resources in a decentralized fashion. In such decentralized settings, selfishness of the users may lead to inappropriate utilization of resources and poor overall performance. When the transmission probabilities of the users are high, collisions dominate the system which in turn degrades the network throughput. In the opposite extreme, when the transmission probabilities are low, there are many idle slots reducing the utilization of the wireless channel. In this context, a recent focus is to design optimization algorithms for contention based networks in the presence of selfish users with each user aiming to maximize its own performance [1], [2].

In the presence of selfish wireless nodes, game theory appears as a natural modeling framework, since it provides incentive compatibility to optimization, and networking problems. In this paper, we propose a noncooperative game that achieves a steady state with desirable features such as finite and stable user backlogs. In this game, the strategy of a wireless user is the selection of its channel access probability, and the cost of this strategy is a function of its utility gain, channel access price, and queue size. The modeling of queue stability within such a game theoretical framework in contention based wireless networks constitutes as one of the contributions of our work. We characterize the equilibrium point of the proposed game and investigate its existence and uniqueness properties. Furthermore, we study dynamical

and distributed algorithms for computing and achieving the equilibrium solution. Specifically, we propose a gradient-based algorithm and prove its convergence to the equilibrium.

It may seem that selfish users applying their own strategies could lead to poor performance by constantly colliding in an attempt to maximize their individual throughput. However, the system performance depends on the cost associated with the users' transmissions [3]. Specifically, we show that the stability of user queues can be guaranteed by changing values of the linear channel access prices to eliminate queue drifts.

In this paper, we design a noncooperative game in a slotted ALOHA setting, and analyze the existence and uniqueness of the Nash Equilibrium (NE) solution. The convergence of a gradient based algorithm to this equilibrium is also proven. Finally, we obtain an implicit relationship between the individual channel access prices of the users that needs to be satisfied for the stability of user queues.

II. RANDOM ACCESS GAME

We consider an uplink wireless network where N users are competing to gain access to a single base station (BS). The time is slotted, and the length of the time slot is equal to the channel coherence interval. The achievable transmission rate of user i at time slot t is $C_i(t)$. The medium access mechanism is basic slotted ALOHA [4], where each user i attempts to transmit with some probability $q_i(t)$. ALOHA is a medium access mechanism which is sufficiently simple to analyze but at the same time that is sufficiently generic to draw meaningful conclusions. Also, some form of ALOHA protocol is used in many different advanced wireless access protocols, e.g., cellular networks.

In this work, we assume that the network consists of selfish users where each user aims to maximize its net benefit while keeping its queue stable at the same time. The net benefit of a user is defined as the difference between the utility obtained from the network access and the cost of this access. The utility function is taken to be a non-decreasing concave function of the throughput. This choice is of practical interest, since a small increase in the rate in the low rate regime is generally more appreciated than a small increase in the high rate regime. In accordance with most prior works, from now on, we assume a logarithmic utility function.

When interactions between the users are taken into account, game theory emerges as a natural modeling framework. In this paper, we design a noncooperative game model, in which users not only aim to maximize their utilities but

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also minimize their queue backlogs. The base station participates in the game by controlling the price of wireless medium access attempts. The players of the game are the users of the network, and their actions are defined by their transmission probabilities, q_i . We assume that each participating user is active, i.e. transmits at least with a very small but nonzero probability ϵ . At each time slot t , the following game is played.

Definition 1. *The stabilizing random access game in slotted ALOHA system is defined as*

$$\Gamma \triangleq \{\mathcal{N}, (s_i)_{i \in \mathcal{N}}, (J_i)_{i \in \mathcal{N}}\}, \quad (1)$$

where \mathcal{N} is the set of N wireless users in the network; $s_i \triangleq \{q_i : q_i \in [\epsilon, 1]\}$ is the strategy space corresponding to transmission probabilities of users; and J_i is the cost of strategy of user i .

Let us define the user cost function as the sum of three terms, where each term identifies an important aspect of the system model. The first term is the utility achieved by the user and is assumed to be related to the log throughput, i.e.,

$$J_i^1(t) = -\eta_i \log \left[C_i(t) q_i(t) \prod_{j \neq i} (1 - q_j(t)) \right]. \quad (2)$$

In (2) $\eta_i > 0$ represents the preference of the different types of users for network throughput¹.

The second term penalizes the positive drift in the queue size, and it is utilized to achieve the stability of user queues.

$$J_i^2(t) = \Delta B_i(t). \quad (3)$$

In (3), $\Delta B_i(t)$ is defined as the drift in queue size of user i which is obtained by subtracting the service rate from the arrival rate:

$$\Delta B_i(t) = A_i(t) - C_i(t) q_i(t) \prod_{j \neq i} (1 - q_j(t)),$$

where $A_i(t)$ is the arrival rate of packets for user i at time t .

The final term represents the cost of access to the channel, and it can be interpreted as the punishment of the greedy behavior:

$$J_i^3(t) = k_i q_i(t). \quad (4)$$

In (4), $k_i > 0$ is the linear price of channel access attempt.

Overall, we have the following aggregate cost function:

$$J_i(q_i, q_{-i}) = J_i^1(t) + J_i^2(t) + J_i^3(t). \quad (5)$$

We are going to first analyze the game in every slot, and hence we omit the time parameter t for brevity. Given the strategy vector, \mathbf{q}_{-i} of all other users, i.e., $\mathbf{q}_{-i} \triangleq (q_1, \dots, q_{i-1}, q_{i+1}, \dots, q_N)$, each user i aims to solve the following optimization problem:

$$\min_{q_i} J_i(q_i, \mathbf{q}_{-i}) \quad (6)$$

¹The negative sign here is due to the cost minimization convention adopted in this paper as opposed to utility maximization.

We note that the players (users) are tightly coupled with each other in the sense that the actions (transmission probabilities) of each player affect the performance of others significantly. In fact, a single player can even block access to whole channel single-handedly if $q = 1$. Unsurprisingly, overall throughput of users can only be achieved if the transmission probabilities decrease proportional to the number of users sharing the channel. The selfish nature of the players may prevent obtaining (such) a mutually beneficial solution by themselves as a result of a phenomenon well-known as “tragedy of commons”, i.e. the (Nash) equilibrium outcome of the game being very undesirable. Pricing schemes are utilized in such cases to improve the outcome of the game. In the next section, the (Nash equilibrium) solution of the game and its properties are analyzed.

III. EQUILIBRIUM AND STABILITY ANALYSIS

A. Existence and Uniqueness of Nash Equilibrium

One of the fundamental issues in the random access game is the analysis of equilibrium solutions, especially existence and uniqueness of a Nash equilibrium (NE) solution. The definition of the NE is provided below for completeness.

Definition 2. *The strategy vectors, i.e. transmission probabilities of wireless users, \mathbf{q}^* , of the random access game Γ defined in (1) is said to be in Nash Equilibrium if no user can improve its cost function by deviating from Nash Equilibrium point [5]. In other words,*

$$J_i(q_i^*, \mathbf{q}_{-i}^*) \leq J_i(q_i, \mathbf{q}_{-i}^*), \quad \forall q_i \in s_i. \quad (7)$$

In game Γ , since the user cost function is convex, the NE solution may exist at the intersection of the player best-responses [5], which follow from the first-order optimality condition:

$$\begin{aligned} \frac{\partial}{\partial q_i} J_i(q_i^*, \mathbf{q}_{-i}^*) &= 0, \quad \forall i \\ \Rightarrow k_i - C_i \prod_{j \neq i} (1 - q_j^*) - \frac{\eta_i}{q_i^*} &= 0, \quad \forall i. \end{aligned} \quad (8)$$

Hence, the NE transmission probability, q_i^* , is

$$q_i^* = \left[\frac{\eta_i}{k_i - C_i \prod_{j \neq i} (1 - q_j^*)} \right]_{\epsilon}^1, \quad (9)$$

where $[\cdot]_{\epsilon}^1 = \min(\max(\epsilon, \cdot), 1)$, for some $\epsilon > 0$, i.e., the value is bounded above and below, respectively.

As described in (9), the transmission probability at the equilibrium of user i decreases in the linear price, k_i , and the transmission probability of other users, q_j^* , $\forall j \neq i$. On the other hand, an increase in the achievable transmission rate, C_i , results in an increase in q_i^* .

Next, we analyze the existence and uniqueness of the Nash Equilibrium. Our analysis applies the results given in [6] and [7] for our particular game.

Lemma 1. *The strategy space of the game Γ , defined in (1), $\mathcal{Q} = [\epsilon, 1]^N \subset \mathbb{R}^N$, is convex, compact, and has a nonempty interior, provided that $\epsilon < 1$.*

Lemma 2. *The cost function of the i^{th} player, J_i in (5), is twice continuously differentiable and strictly convex in q_i , i.e., $\partial^2 J_i / \partial q_i^2 > 0$, and $\partial^2 J_i / \partial q_i \partial q_j > 0$ on \mathcal{Q} .*

Proof Note that $\eta_i > 0$ and $k_i > 0$. The second partial derivatives of cost function, J_i , are $\frac{\partial^2 J_i}{\partial q_i^2} = \frac{\eta_i}{q_i^2}$, and $\frac{\partial^2 J_i}{\partial q_i \partial q_j} = C_i \prod_{n \neq i, j} (1 - q_n)$. Thus, $\frac{\partial^2 J_i}{\partial q_i^2}$ and $\frac{\partial^2 J_i}{\partial q_i \partial q_j}$ are always greater than zero on \mathcal{Q} . ■

Proposition 1. *The random access game, Γ defined in (1), admits at least one NE solution.*

Proof According to Lemma 1 and 2, J_i is a differentiable convex function. Hence, $\frac{\partial J_i}{\partial q_i}$ is a continuous function, and so is its inverse $\left(\frac{\partial J_i}{\partial q_i}\right)^{-1}$. Note that the range of transmission probabilities, $[0, 1]$, is a connected and compact set. Therefore, the range of $\left(\frac{\partial J_i}{\partial q_i}\right)^{-1}$ is also a connected and compact set. Based on Theorem 4.4 in [6] and Theorem 1 in [7], we can conclude that there exists at least one NE solution. ■

Next, we investigate the conditions under which the game admits a unique NE solution. Let $g(\mathbf{q}) = [g_1(\mathbf{q}), \dots, g_N(\mathbf{q})]$ where $g_k(\mathbf{q}) = \frac{\partial J_k}{\partial q_k}$, $k = 1, \dots, N$ and $G(\mathbf{q})$ be the Jacobian of $g(\mathbf{q})$ which is defined as:

$$G(\mathbf{q}) = \begin{bmatrix} b_1 & a_{12} & \dots & a_{1N} \\ a_{21} & b_2 & \dots & a_{2N} \\ \vdots & & \ddots & \vdots \\ a_{N1} & a_{N2} & \dots & b_N \end{bmatrix}, \quad (10)$$

where $b_i = \partial^2 J_i / \partial q_i^2$ and $a_{ij} = \partial^2 J_i / \partial q_i \partial q_j$.

Definition 3. *A matrix is said to be strictly diagonally dominant if in every row of the matrix, the magnitude of the diagonal entry in that row is larger than the sum of the magnitudes of all the other (non-diagonal) entries in that row.*

In our case, the matrix $G(\mathbf{q})$ is strictly diagonally dominant if $|b_i| > \sum_{j \neq i} |a_{ij}|$, $\forall i$.

Lemma 3. *If $q_i < \sqrt{\frac{\eta_i}{C_i(N-1)}}$, $\forall i$, then $G(\mathbf{q})$ is strictly diagonally dominant.*

Proof

$$q_i < \sqrt{\frac{\eta_i}{C_i(N-1)}} \quad (11)$$

$$\frac{\eta_i}{q_i^2} > C_i(N-1) \quad (12)$$

$$\frac{\eta_i}{q_i^2} > \sum_{j \neq i} \left(C_i \prod_{n \neq i, j} (1 - q_n) \right), \quad (13)$$

$$b_i > \sum_{j \neq i} a_{ij}, \quad (14)$$

where (13) follows from $(1 - q_n) \leq 1$, and (14) is from the proof of Lemma 2. ■

The following result, which is a variation of Theorem 2.1 in [8], is also needed to further the analysis.

Lemma 4. [9] *A strictly diagonally dominant matrix is non-singular.*

We next establish the uniqueness of an **inner** NE solution on a subset of the strategy space \mathcal{Q} :

Theorem 1. *The random access game Γ of Definition 1 admits an inner Nash equilibrium solution that is unique on the strategy space*

$$\bar{\mathcal{Q}} := \bigotimes_{i=1}^N \left(\epsilon, \sqrt{\frac{\eta_i}{C_i(N-1)}} \right), \quad (15)$$

where \bigotimes denotes cross-product of the interval sets.

Proof Suppose that there are two inner equilibrium points, represented by \mathbf{q}^1 and \mathbf{q}^0 . Define the strategy vector $\mathbf{q}(\theta)$ as a convex combination of the two equilibrium points $\mathbf{q}^1, \mathbf{q}^0$:

$$\mathbf{q}(\theta) = \theta \mathbf{q}^1 + (1 - \theta) \mathbf{q}^0, \quad (16)$$

where $0 < \theta < 1$. Note that $\mathbf{q}(\theta)$ still satisfies the condition in the theorem. When we take the derivative of $g(\mathbf{q}(\theta))$ with respect to θ , we obtain

$$\frac{dg(\mathbf{q}(\theta))}{d\theta} = G(\mathbf{q}(\theta)) \frac{d\mathbf{q}(\theta)}{d\theta} = G(\mathbf{q}(\theta)) (\mathbf{q}^1 - \mathbf{q}^0). \quad (17)$$

By integrating (17) over θ yields,

$$g(\mathbf{q}^1) - g(\mathbf{q}^0) = \left[\int_0^1 G(\mathbf{q}(\theta)) d\theta \right] (\mathbf{q}^1 - \mathbf{q}^0). \quad (18)$$

Recall that \mathbf{q}^1 and \mathbf{q}^0 are equilibrium points, so $g(\mathbf{q}^1) = 0$ and $g(\mathbf{q}^0) = 0$. From Lemma 3, we know that $G(\mathbf{q}(\theta))$ is strictly diagonally dominant, and hence, $\int_0^1 G(\mathbf{q}(\theta)) d\theta$ is strictly diagonally dominant and it is also non-singular based on Lemma 4. Thus, the matrix, $\int_0^1 G(\mathbf{q}(\theta)) d\theta$, is also non-singular. Then, it is clear that (18) is equal to zero, only when $\mathbf{q}^1 - \mathbf{q}^0 = 0$. Therefore, there cannot be more than one equilibrium point. ■

Next, we include the **boundary solutions** of the game in our investigation:

Theorem 2. *The random access game Γ of Definition 1 admits a unique inner Nash equilibrium solution on the strategy space*

$$\tilde{\mathcal{Q}} := \bigotimes_{i=1}^N \left[\epsilon, \sqrt{\frac{\eta_i}{C_i(N-1)}} \right], \quad (19)$$

if ϵ is chosen sufficiently small and

$$k_i > C_i + \sqrt{C_i \eta_i (N-1)} \quad \forall i.$$

Proof Notice that for a sufficiently small ϵ the players can always improve their performance by increasing their transmission rate when they transmit at the lower boundary, which follows directly from (8). Likewise, the sufficient condition on individual prices k_i ensures that $\partial J_i / \partial q_i > 0$ at the upper boundary points. Thus, any boundary solution on $\bar{\mathcal{Q}}$ cannot constitute a Nash equilibrium solution (by definition of NE). The rest of the proof follows directly from the one of Theorem 1. ■

B. Convergence to Nash Equilibrium under Gradient Algorithm

Once we establish that there is a unique Nash equilibrium solution under certain conditions, the next question is to determine a distributed algorithm achieving this solution, since the unique Nash equilibrium solution given in (8) cannot be explicitly found.

We consider a dynamic transmission probability update mechanism, where each user utilizes a gradient algorithm to solve its own optimization problem within a given time slot. Let us assume that user i updates its transmission probability according to the following dynamic equation:

$$\frac{d q_i}{d t} = -\frac{\partial J_i}{\partial q_i} =: \phi_i, \quad (20)$$

where $\phi_i = (-k_i + C_i \prod_{j \neq i} (1 - q_j) + \frac{\eta_i}{q_i})$.

The dynamic system in (20) defines a method for user i to update its transmission probability based on the channel capacities and the transmission probabilities of the other users in the system. This update is in the opposite direction of the gradient of the total cost.

Note that both the users and the base station has sufficient information to implement the algorithm. The users need to measure their own throughput in order to update their transmission probabilities. Likewise, by measuring the throughputs of individual users, the base station can compute their transmission probabilities without asking them explicitly, and hence impose prices. The only information exchange, other than measurements, in the system is due to base station telling users their individual prices k_i .

We next show that the dynamic update mechanism described by (20) is asymptotically stable, and hence converges to the unique inner NE of the game.

Theorem 3. *Let $\mathbf{q}^{NE} \triangleq [q_1^*, q_2^*, \dots, q_N^*]$ be the unique inner NE of the game Γ , defined in (1), with the strategy space $\tilde{\mathcal{Q}}$ in (19). The system dynamics stated in (20) are asymptotically stable, and converge to the unique NE, under the sufficient conditions of Theorem 2, i.e., if ϵ is chosen sufficiently small and*

$$k_i > C_i + \sqrt{C_i \eta_i (N - 1)} \quad \forall i.$$

Proof

Let us introduce a candidate quadratic Lyapunov function, V , defined as,

$$V \triangleq \frac{1}{2} \sum_i \phi_i^2. \quad (21)$$

Note that since there is a unique equilibrium solution, \mathbf{q}^* , $V = 0$, if and only if $\mathbf{q} = \mathbf{q}^*$ and $V > 0$ for all $\mathbf{q} \neq \mathbf{q}^*$, i.e., the equilibrium point corresponds to the lowest energy state of the Lyapunov function defined in (21).

Next, we need to show that Lyapunov function is a decreasing function for all values of $\mathbf{q} \neq \mathbf{q}^*$. However, we

first need to calculate the first derivative of ϕ_i as:

$$\frac{d \phi_i}{d t} = \frac{d^2 q_i}{d t^2} = - \sum_{j \neq i} \phi_j a_{ij} - \frac{\eta_i}{q_i^2} \phi_i, \quad (22)$$

where $a_{ij} = C_i \prod_{n \neq i, j} (1 - q_n)$ and $\phi_j = \frac{d q_j}{d t}$.

In order for V to be a decreasing function, its first derivative should always be negative:

$$\begin{aligned} \frac{d V}{d t} &= \sum_i \phi_i \frac{d \phi_i}{d t}, \quad (23) \\ &\stackrel{(a)}{=} - \left(\sum_i \frac{\eta_i}{q_i^2} \phi_i^2 + \sum_i \sum_{j \neq i} a_{ij} \phi_j \phi_i \right), \\ &\stackrel{(b)}{=} - \left(\sum_i \frac{\sqrt{\eta_i}}{q_i} \phi_i \right)^2 - \sum_i \sum_{j > i} |\phi_i \phi_j| \left(-a_{ij} - a_{ji} + \frac{2\sqrt{\eta_i \eta_j}}{q_i q_j} \right), \\ &\stackrel{(c)}{\leq} - \left(\sum_i \frac{\sqrt{\eta_i}}{q_i} \phi_i \right)^2 < 0, \quad \forall \mathbf{q} \neq \mathbf{q}^*, \quad (24) \end{aligned}$$

where (a) is obtained by inserting (22) into (23), (b) follows from the square completion, and (c) follows by imposing the assumption, $a_{ij} + a_{ji} < \frac{2\sqrt{\eta_i \eta_j}}{q_i q_j}$. Note that, in (a), when $\phi_i \phi_j > 0$, $\frac{d V}{d t}$ is negative. Hence, in (b), we consider only the case, when $\phi_i \phi_j$ is negative.

Thus, the system is stable under the assumption of $a_{ij} + a_{ji} < \frac{2\sqrt{\eta_i \eta_j}}{q_i q_j}$. Now we investigate the conditions on C_i and k_i that realize this assumption. We claim that when $k_i + \frac{\sqrt{\eta_i \eta_j}}{2} > C_i$, then the above condition is satisfied. Furthermore, this condition is superseded by the sufficient condition for the uniqueness of the inner NE in Theorem 2.

$$\begin{aligned} &2 \left(k_i + \frac{\sqrt{\eta_i \eta_j}}{2} - C_i \right) \left(k_j + \frac{\sqrt{\eta_i \eta_j}}{2} - C_j \right) \stackrel{(a)}{>} 0 \\ &2(k_i - C_i)(k_j - C_j) - \sqrt{\eta_i \eta_j}(C_i + C_j) \stackrel{(b)}{>} 0 \\ &\frac{2}{\sqrt{\eta_i \eta_j}} \left(k_i - C_i \prod_{n \neq i} (1 - q_n) \right) \left(k_j - C_j \prod_{m \neq j} (1 - q_m) \right) \stackrel{(c)}{>} \\ &\quad \prod_{l \neq i, j} (1 - q_l)(C_i + C_j) \\ &\quad \frac{2\sqrt{\eta_i \eta_j}}{q_i q_j} \stackrel{(d)}{>} a_{ij} + a_{ji}. \end{aligned}$$

where (a) follows from the condition, $k_i + \frac{\sqrt{\eta_i \eta_j}}{2} - C_i > 0$, (b) follows from $(k_i + k_j)\sqrt{\eta_i \eta_j} + \frac{\eta_i \eta_j}{2} > 0$ and (c) follows from $k_i - C_i \prod_{n \neq i} (1 - q_n) > k_i - C_i$ and $\prod_{l \neq i, j} (1 - q_l)(C_i + C_j) < C_i + C_j$. In (d), we use the definitions of q_i and a_{ij} . Therefore, the gradient update algorithm converges asymptotically to the unique NE of the game. ■

IV. STABILITY ANALYSIS OF USER QUEUES

In this section, we investigate how the pricing parameter, k_i should be chosen so that the user queues are stabilized. We obtain an implicit relationship between k_i 's that needs to be satisfied to ensure that queue drift in each queue is zero, i.e., $\Delta B_i^* = 0$, when each user has potentially different set of system parameters and costs. Secondly, we obtain closed form solution of k for symmetric two user case, where the

users are indistinguishable in terms of their cost and system parameters.

In order for a queue to be stable at the equilibrium, the drift of the queue should be negative or zero, i.e.,

$$\Delta B_i^* = A_i - C_i q_i^* \prod_{j \neq i} (1 - q_j^*) \leq 0. \quad (25)$$

If the above is satisfied with equality, the queue size neither decreases nor increases. In this case, one may assume that there is always a packet in the queue. Here, we give analysis when $\Delta B_i^* = 0$.

Inserting equilibrium solution, q_i^* , in (9) into the drift equation in (25), we obtain

$$\prod_{j \neq i} (1 - q_j^*) = \frac{A_i k_i}{(A_i + \eta_i) C_i}. \quad (26)$$

Let $\alpha_i = \log(1 - q_i^*)$ and $\beta_i = \log\left(\frac{A_i k_i}{(A_i + \eta_i) C_i}\right)$. Then, from (26), we obtain $\sum_{j \neq i} \alpha_j > \beta_i$. Note that the transmission probabilities vary with respect to k , which is the variable used to punish the greedy behavior. Also as shown in (26), the value of k affects the stability of the queues.

Let us define matrix E as a matrix with all entries except diagonal ones are equal to 1, and diagonal entries are zero. Also, define $d = [\alpha_1, \alpha_2, \dots, \alpha_N]$ and $b = [\beta_1, \beta_2, \dots, \beta_N]$. Then, (26) can be re-written in matrix form as:

$$\begin{aligned} E \cdot d^T &= b^T \\ d^T &= E^{-1} \cdot b^T. \end{aligned} \quad (27)$$

Note that E^{-1} is a matrix in which diagonal entries are $\frac{2-N}{N-1}$, and other entries are $\frac{1}{N-1}$.

Since (26) should be satisfied so that the user queues are stable, the i^{th} row of $E^{-1} \cdot b^T$ should be equal to the i^{th} row of d^T :

$$\alpha_i = \frac{1}{N-1} \left(\beta_i(2-N) + \sum_{j \neq i} \beta_j \right). \quad (28)$$

After some mathematical manipulations, we obtain the following equation of the equilibrium transmission probability that ensures the stability of the queues as:

$$q_i^* = 1 - \left(\frac{A_i k_i}{(A_i + \eta_i) C_i} \right)^{\frac{2-N}{N-1}} \prod_{j \neq i} \left(\frac{A_j k_j}{(A_j + \eta_j) C_j} \right)^{\frac{1}{N-1}} \quad (29)$$

Unfortunately, the closed form solution of k_i cannot be obtained from (29) due to the nonlinear structure of the inequality.

Aforementioned analysis can be extended to the case when the queue drifts are negative. When the queue drifts are negative, the user queues tend to get empty. Hence, some of the users do not have sufficient number of packets to transmit and it is not possible to guarantee an inner point Nash equilibrium solution. In fact, those users with empty queues are no longer part of the game, since they cannot transmit with positive probability. As users with empty queues are

out of the game, the game is played only among those players with non-empty queues. This new game can be analyzed in exactly the same way as discussed before, but obviously the game has fewer number of players. In this case, both the Nash equilibrium transmission probabilities and the access prices for ensuring negative queue drift need to be re-calculated according to our results given in earlier sections. As the queues get empty and get filled up again, the whole analysis have to be repeated.

Note that, the queues in the system cannot be stabilized for all arrival rates. Thus, a closed form solution can give us intuition through which arrival rates, A , and prices, k , the stability of the queues can be realized. Thus, we now consider two user symmetric case, i.e., $A_1 = A_2 = A$, $C_1 = C_2 = C$ and $\eta_1 = \eta_2 = \eta$.

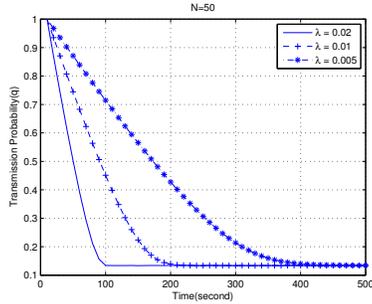
By solving (29) and equilibrium solution in (9) simultaneously, we obtain the following prices, k , which drives queue drift to zero:

$$k = \frac{0.5(C \mp \sqrt{C^2 - 4AC})(A + \eta)}{A}. \quad (30)$$

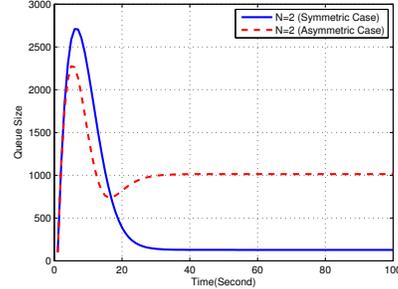
It is easy to observe that the user queues are stable only for $A < \frac{C}{4}$. For symmetric case, the maximum rate is achieved when $q^* = 1/2$, which corresponds to a maximum achievable rate of $C/4$. Thus, the condition $A < \frac{C}{4}$ suggests that unless the arrival rate, A , is smaller than the maximum achievable rate, the queues cannot be stabilized.

V. NUMERICAL ANALYSIS

The game theoretical framework and the resulting transmission probability update algorithms are analyzed numerically in MATLAB. We first investigate the rate of convergence of the dynamic system in (20) for varying values of step size λ_i . For this purpose, we consider a network with 50 users. The value of channel capacity is uniformly randomly chosen in $[0, 10]$. The maximum achievable rate is 10 bits/channel use when the signal-to-noise-ratio (SNR) is 30dB which is considered as an upper limit on SNR in the literature. A slot time is taken to be 100 microseconds. In the experiments, we observe that throughput converges in approximately 10^5 slots which corresponds to 10 seconds, when the number of users, N , is equal to 50. Thus, users wait for 10 seconds corresponding to 10^5 slots before updating their transmission probabilities. Meanwhile, the pricing parameter k takes values uniformly randomly in $[1, 20]$. In addition, we assume that the preferences of the users towards channel access, i.e., η_i are identical, and equal to one. We have performed the simulations for three different step sizes ($\lambda = 0.02, 0.01, 0.005$), and plot the transmission probabilities for a randomly selected user in Figure 1a. Note that when the step size, λ , is equal to 0.02, the convergence is faster. However, after convergence, the transmission probability slightly oscillates. This oscillation is due to the fact that the continuity assumption for discrete variables is violated for large step sizes. The simulation is repeated for different number of users, and the results are similar to those presented in Figure 1a. Furthermore, we run the simulations for different values of user preferences, η ,



(a) Transmission probability



(b) Queue sizes.

Fig. 1. Numerical results

and the convergence rate remains approximately the same for different values of η .

We next investigate the stability of user queues for the two-user symmetric and asymmetric cases. For the symmetric case, in which the system parameters are the same for all users, we select k as in (30). The capacity, C , is equal to 0.5, and the packet arrival rate is 0.1. For asymmetric case, in which users have different set of system parameter. We solve (29) numerically to obtain the value of k_i necessary to ensure the stability of user queues. The values of C_i and A_i are uniformly randomly selected in the intervals $[0, 10]$ and $[0, 1]$; however, we also ensure that the queues can be stabilized with the selected values. The step size is selected to be $\lambda = 0.01$ for both cases. For the convergence of throughput, users wait for 1 second, since throughput converges more rapidly for small number of users. Figure 1b shows that in both cases the queue sizes first fluctuate as the algorithm converges to the equilibrium solution, \mathbf{q}^* . After that, they show no change confirming theoretical results.

VI. RELATED WORK

A plethora of work have emerged on the issue of optimizing the medium access control mechanism, especially for the slotted ALOHA systems. Here, we restrict ourselves to cite a few that are most closely related to our work, i.e., we focus on optimization of medium access mechanisms using game theory. In [1], a stability region has been obtained for a slotted multi-packet Aloha system with selfish users, perfect information, and under the assumption of some well-known channel models. [2] has considered both the cooperative team problem as well as the noncooperative game problem to minimize the delay in slotted ALOHA. Unlike these works, [10] and [11] have studied distributed choices of transmission probabilities in the slotted Aloha with partial information with imposing priorities and random power. [12] has studied noncooperative equilibria of Aloha networks and their local convergence.

In addition to aforementioned works that focused on the slotted ALOHA medium access mechanism, there are also works investigating other MAC mechanisms. [13] has discussed selfish behavior in CSMA/CA networks using game theoretical approach and proposed a distributed protocol to guide the selfish nodes to a Pareto-optimal Nash equilibrium. [14] has investigated the interaction among wireless nodes in

a game theoretical framework and designed medium access methods that can stabilize the network around a steady state with a target fairness and high efficiency.

VII. CONCLUSION

We have studied a noncooperative game among the users of a contention-based wireless network. The outcome of the game stabilizes the user queues based on the choice of pricing parameters. We have characterized the Nash equilibrium of the game and investigated the convergence properties of a distributed gradient update algorithm to compute the Nash equilibrium. In addition, we have shown that we can move the equilibrium point to desirable regions characterized by the stability of user queues.

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