A Hybrid Noncooperative Game Model for Wireless Communications

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Abstract

We investigate a hybrid noncooperative game motivated by the practical problem of joint power control and base station (BS) assignment in code division multiple access (CDMA) wireless data networks. We model the integrated power control and BS assignment problem such that each mobile user's action space includes not only the transmission power level but also the respective BS choice. Users are associated with specific cost functions consisting of a logarithmic user preference function in terms of service levels and convex pricing functions to enhance the overall system performance by limiting interference and to preserve battery energy. We study the existence and uniqueness properties of pure strategy Nash equilibrium solutions of the hybrid game, which constitute the operating points for the underlying wireless network. Since this task cannot be accomplished analytically even in the simplest cases due to the nonlinear and complex nature of the cost and reaction functions of mobiles, we conduct the analysis numerically using grid methods and randomized algorithms. Finally, we simulate a dynamic BS assignment and power update scheme, and compare it with "classical" noncooperative power control algorithms in terms of aggregate SIR levels obtained by users.

Keywords: *Noncooperative games, Nash equilibrium, hybrid games, wireless networks, power control.*

1 Introduction

The primary objective of mobile users in wireless networks is to achieve and maintain a satisfactory level of service, which may be described in terms of signal-to-interference ratio (SIR). The mobiles may vary their uplink transmission power levels and connections to the base stations in order to reach this goal. Since in code division multiple access (CDMA) systems signals of other users can be modeled as interfering noise signals, each mobile degrades the level of service of others as a result of these decisions. Hence, the joint power control and base station (BS) assignment problem [1] leads to a conflict of interest between the individual

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users. In addition, if users have different preferences for the level of service or have varying SIR requirements, then the power control problem can be posed as one of resource allocation. Furthermore, under a distributed power control regime, the mobiles cannot have detailed information on each other's preferences and actions due to communication constraints inherent to the system. It is therefore appropriate to address the joint power control and BS assignment problem within a noncooperative game theoretic framework, where Nash equilibrium (NE) [2] provides a relevant solution concept.

Several studies exist in the literature that use noncooperative game theoretic schemes to address the power control problem, including the ones by the authors [3–5]. In all of the previous studies, mobiles were considered to be connected to the closest BS, i.e. the one with the lowest channel attenuation. However, the service or SIR level of a mobile is also affected by the number of mobiles in the vicinity of the same BS. Therefore, the performance of the mobiles can be improved by introducing the choice of BS as an additional discrete decision variable in addition to the positive real power level variable. In a related study, Saraydar et al. [6] consider a similar problem. However, they analyze the problem in two separate parts involving BS choice and optimization of the power level, and hence, do not model it as a hybrid game.

Accordingly, we consider in this paper a hybrid noncooperative game motivated by the practical problem of joint power control and BS assignment in CDMA wireless data networks. This constitutes an extension on the model in our earlier studies [3–5] with an additional degree of freedom in optimization. Specifically, we model the integrated power control and BS assignment problem in such a way that each mobile's action space includes not only the transmission power level but also the choice of the BS. The outcomes of the user actions are reflected in a specific cost structure, and each user is associated with a cost function that is parametrized by user specific prices. The convex pricing schemes considered aim to enhance the overall system performance by limiting the interference and preserve battery energy. We investigate the existence and uniqueness of pure strategy NE solutions of the hybrid game, which constitute the operating points for the underlying wireless network. We make the inherent assumption that a mobile is connected to a single BS. Furthermore, mixed strategy solutions are not feasible for the system considered due to the prohibitive handoff costs in the implementation. We therefore focus on pure strategy solution. Unfortunately, conditions for the existence and uniqueness of pure NE cannot be derived analytically even in the simplest cases due to the nonlinear and complex nature of cost and reaction functions of mobiles. Hence, we investigate solutions of this interesting hybrid game numerically using grid methods and randomized algorithms.

The next section describes the wireless network model adopted. We define the hybrid power control game and the cost function in Section 3. Section 4 discusses Nash equilibrium solution and contains Subsection 4.1 which describes randomized algorithms for numerical analysis. We present our simulation results in Section 5 where we investigate the existence and uniqueness properties of Nash equilibrium solutions in Subsection 5.1, and analyze a power update and BS assignment scheme in Subsection 5.2. The paper concludes with a recap of the results and elucidation of directions for future research in Section 6.

2 The Wireless Network Model

We consider a multicell CDMA wireless network model similar to the ones described in [3–5]. The system consists of a set $\mathcal{B} := \{1, \ldots, N\}$ of base stations and a set $\mathcal{M} := \{1, \ldots, M\}$ of users. The number of users on the network is limited through an admission control scheme. The i^{th} mobile transmits with a nonnegative uplink power level of $p_i \leq p_{max}$, where p_{max} is an upper-bound imposed by physical limitations of the mobiles. Figure 1 depicts a simplified picture of the wireless network model considered. In this paper, we investigate the case where mobiles are given the freedom of choosing the BS connection in addition to determining their transmission power levels. Hence, each mobile connects to a BS which it chooses from the set of BSs on the network, $\mathcal{B}$.

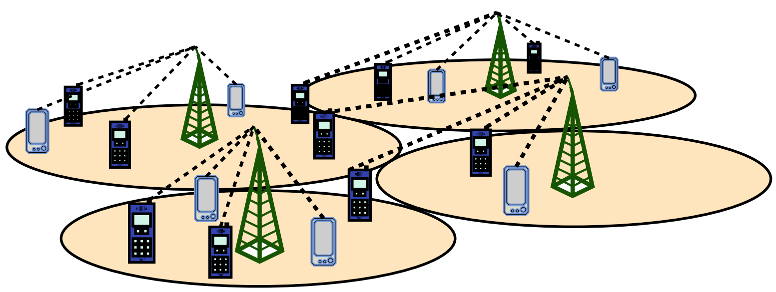

Figure 1: An abstract diagram of the CDMA wireless network model.

We define $h_{il}p_i$ as the instantaneous received power level from user i at the l^{th} BS. We assume that a mobile connects to one BS only at any given time. The quantity h_{il} $(0 < h_{il} < 1)$ represents the channel gain between the i^{th} mobile and the l^{th} BS [7]. Ignoring fast-time scale fading (such as Rayleigh fading) and shadowing effects in order to simplify the analysis, we model h_{il} in such a way that it depends only on the location of the mobiles with respect to the base stations. Hence, the channel gain h_{il} is given by

$$h_{il} := \left(\frac{0.1}{d_{il}}\right)^2, \tag{1}$$

where d_{il} denotes the (Euclidean) distance of the mobile to the BS, and the loss exponent is chosen as 2, which corresponds to a free space environment [7]. In addition, we assume that the location of a mobile, which is the main factor affecting the channel gain, h, does not change significantly over the time scale of this analysis. This assumption is justified by the fact that the power control algorithm operates with a high frequency.

The level of service a mobile receives is described in terms of SIR [3, 4]. In accordance with the interference model considered, the SIR obtained by mobile i at the base station l is given by

$$\gamma_{il} := \frac{L h_{il} p_i}{\sum_{j \neq i} h_{jl} p_j + \sigma_l^2}. \tag{2}$$

Here, $L := W/R > 1$ is the spreading gain of the CDMA system, where W is the chip rate and R is the data rate of the user. Furthermore, the additional interference at the BS l due to factors other than the transmissions of other mobiles is modeled as a fixed background noise, of variance σ_l^2.

3 The Hybrid Power Control Game

We consider a power control game where each user (mobile) is associated with a specific cost function. Since a user can choose both its power level, which is a continuous variable, and the BS it connects, which is discrete in nature, we call this a *hybrid power control game*. The action space, S_i of the i^{th} user is then defined as

$$S_i = \{(b, p) : b \in \mathcal{B} = \{0, 1, \ldots, N\} \text{ , and } p \in [0, p_{max}]\}, \tag{3}$$

and the actions are denoted by $s_i := (b_i, p_i)$. The cost function, J_i, of user i is defined as the difference between the utility function of the user and its pricing function, $J_i = P_i - U_i$. The utility function, $U_i(\mathbf{p}, b_i)$, is chosen as a logarithmic function of the i^{th} user's SIR, which we denote by $\gamma_i(\mathbf{p}, b_i)$. It quantifies approximately the demand or *willingness to pay* of the user for a certain level of service. This utility function can also be interpreted as being proportional to the Shannon capacity [7], if we make the simplifying assumption that the noise plus the interference of all other users constitute an independent Gaussian noise. This means that this part of the utility is simply linear in the throughput that can be achieved (or approached) by user i using an appropriate coding, as a function of its transmission power [3].

The pricing function, $P_i(p_i)$, on the other hand, is imposed by the system to limit the interference created by the mobile, and hence to improve the system performance [6]. At the same time, it can also be interpreted as a cost on the battery usage of the user. It is a convex function of p_i, the power level of the user. Accordingly, the cost function of the i^{th} user is defined as

$$J_i(\mathbf{p}, b_i) = P_i(p_i) - \log(1 + \gamma_i(\mathbf{p}, b_i)), \tag{4}$$

where $\gamma_i(\mathbf{p}, b_i)$ is the i^{th} mobile's SIR level under the given vector of power levels, $\mathbf{p}$, of all mobiles and its BS choice, b_i. One possible pricing function is the linear one, $P_i(p_i) = \lambda_i p_i$, where λ_i is a user-specific parameter. This structure will be used extensively, though not exclusively, in the paper.

In the hybrid power control game defined, the i^{th} user's optimization problem is to minimize its cost (4), given the sum of power levels of all other users as received at the base stations. Thus, the reaction function of user i is

$$p_i(b_i, \mathbf{p}) = \arg \min_{\mathrm{s_i=(b_i,p_i)}} \mathrm{J_i(b_i, \mathbf{p})}. \tag{5}$$

Furthermore, if the mobile is connected to the l^{th} BS, i.e. $b_i = l,\ l \in \mathcal{B}$, and P_i is chosen as linear in p_i, $P_i = \lambda_i p_i$, then the optimal power level is given by

$$p_i(b_i = l,\, \mathbf{p}) = \begin{cases} \dfrac{1}{\lambda_i} - \dfrac{1}{L h_{il}} \left(\displaystyle\sum_{j \neq i} h_{jl} p_j + \sigma_l^2 \right), & \text{if} \displaystyle\sum_{j \neq i} h_{jl} p_j < \dfrac{\lambda_i}{L h_{il}} - \sigma_l^2 \\ 0, & \text{else} \end{cases} \tag{6}$$

We note that, given a specific BS, the user's optimization problem admits a unique solution (power level) in the linear pricing case, although it may be a boundary solution. The nonnegativity of the power vector ($p_i \geq 0\ \ , \forall i$) is an inherent physical constraint of the model. We refer to [3] for details on the boundary analysis and conditions for an inner solution in the linear pricing case and [4] for the analysis of more general convex pricing schemes.

The reaction function (5) is the optimal response of the user to the varying parameters in the model. It depends not only on the user-specific parameters like b_i, λ_i, and h_i but also on the network parameter, L, and total power level received at the BS $l\,(= b_i)$ to which the mobile is connected, $\sum_{j=1}^{M} h_{jl} p_j$. Each BS provides the user total received power level using the downlink. We refer to [3,4] for communication constraints and effects. A simplified block diagram of part of the system relevant to the analysis here is shown in Figure 2.

4 Nash Equilibrium

The Nash equilibrium (NE) in the context of the hybrid power control game is defined as a set of actions, s^*, consisting of the BS choice, b^*, power levels, p^*, and corresponding set of costs J^*, of users with the property that no user in the system can benefit by modifying its action while the other players keep theirs fixed. Mathematically speaking, the vector of user actions, $\mathbf{s}^* := (\mathbf{b}^*, \mathbf{p}^*)$, is in NE when s_i^* of any i^{th} user is the solution to the following optimization problem given the equilibrium actions of other mobiles, $\mathbf{s}_{-i}^*$:

$$\min_{s_i} J_i(s_i, \mathbf{s}_{-i}^*). \tag{7}$$

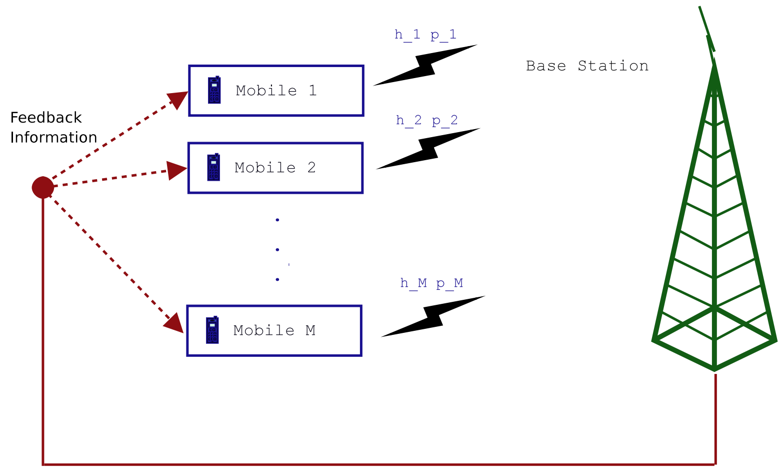

Figure 2: A simplified block diagram of the system.

We have shown in [3, 4] that once mobiles make the decision on BS connections, the power control game defined admits a unique NE under certain conditions. In the hybrid power control game defined, however, it is not possible to establish the existence or uniqueness of NE solution analytically. Therefore, we resort to numerical methods, and *randomized algorithms* [8, 9] stand out as a useful tool for numerical analysis [10].

4.1 Randomized Algorithms and Monte Carlo Methods

We utilize randomized algorithms in order to obtain an estimate on the probabilities of existence and uniqueness of NE solutions in the hybrid power control game. Following the approach in [10], these probability estimates are obtained both through *random sampling* or *gridding* techniques on the parameter space. In this case, the parameter space consists of the locations of the mobiles, user specific parameters such as λ, and system parameters such as L and σ^2. For illustrative purposes, let us call the specific parameter (vector) we are interested in α, while we keep all other parameters fixed.

In the case of random sampling method, the parameter vector α is chosen to be random with given probability density function f_α, having support sets $\mathcal{S}_\alpha$. We can take, for example, $\mathcal{S}_\alpha$ to be the hyper-rectangular set

$$\mathcal{S}_\alpha = \{\alpha : \alpha_i \in [\alpha_i^-, \alpha_i^+] \,,\ i = 1, 2, \ldots, M\} \,, \tag{8}$$

and the density function f_α to be uniform on these sets. That is, for $i = 1, 2, \ldots, M$,

$$f_{\alpha_i} = \begin{cases} \dfrac{1}{\alpha_i^+ - \alpha_i^-} & \text{if } \alpha_i \in [\alpha_i^-, \alpha_i^+] \\ 0 & \text{otherwise} \end{cases} \tag{9}$$

Then, we generate N_α independent identically distributed (i.i.d.) vector samples from the set $\mathcal{S}_\alpha$ according to the density function f_α:

$$\alpha^1, \alpha^2, \ldots, \alpha^{N_\alpha}.$$

Another method for generating the sample points of the parameter is to utilize a gridding technique on the support set $\mathcal{S}_\alpha$. In this case, the samples are generated such that they are evenly spaced on the support set. Since this technique provides a nicely distributed sample set, one might think that random sampling is not necessary. However, gridding techniques suffer from a significant drawback called *curse of dimensionality*. That is, as the dimension of the parameter space increases, the number of samples required to cover the set $\mathcal{B}$ with a uniform grid grows exponentially. Therefore, we resort to random sampling methods when the dimension of the parameter space gets large.

Once the sample points are generated using either random sampling or gridding techniques, we investigate the existence of NE solutions for each sample point or set of parameters. Towards this end, we construct the indicator function

$$\mathcal{I}(\alpha^i) := \begin{cases} 1 & \text{if the game admits a NE} \\ 0 & \text{otherwise.} \end{cases}$$

The estimated probability of existence of a NE is readily given by

$$\widehat{p}_{N_\alpha} = \frac{1}{N_\alpha} \sum_{i=1}^{N_\alpha} \mathcal{I}(\alpha^i) \tag{10}$$

which is equivalent to

$$\widehat{p}_{N_\alpha} = \frac{N_{NE}}{N_\alpha}$$

where N_{NE} is the number of vector samples such that the hybrid game admits a NE. A separate indicator function can be defined in a similar manner to compute the probability of having multiple NE solutions.

In order to obtain a "reliable" probabilistic estimate it is important to know how many samples N_α are needed. Let us denote the real probability of existence of a NE by p_α. The Chernoff bound [8] states that for any $\epsilon \in (0,\, 1)$ and $\delta \in (0,\, 1)$ if

$$N_\alpha \geq \frac{1}{2\epsilon^2} \ln\left(\frac{2}{\delta}\right)$$

then, with probability greater than $1 - \delta$, we have $|\widehat{p}_{N_\alpha} - p_\alpha| < \epsilon$. Note that, this is a problem-independent explicit bound which can be computed *a priori*. We refer to [8, 9, 11] for a detailed discussion on this issue.

5 Numerical Analysis

We first investigate the existence and uniqueness of Nash equilibrium solutions of the hybrid power control game defined in Section 3 numerically on the wireless network described in Section 2. Then, we simulate a dynamic power update and BS assignment scheme for mobiles.

5.1 Existence and Uniqueness of NE

In order to be able to visualize (some of) the results we start with a one-dimensional (1-D) network model where both mobiles and BSs are located along a line. Although we consider this model only for illustrative purposes, it can also be interpreted in such a way that mobiles staying on a road connect to BSs located at the road side. In the first simulation, we analyze two mobiles on a 1-D network of two BSs. The system and user parameters are chosen as $L = 128$, $\sigma_1^2 = \sigma_2^2 = 0.1$, and $\lambda_1 = \lambda_2 = 0.1$. For these sets of simulations a linear pricing scheme, where $P = \lambda_i p_i$, is chosen unless otherwise stated. The mobiles are located using a gridding method such that each sample point is evenly spaced with a distance of 0.125. The locations of the two BSs are chosen to be 0.5 and 1.5, respectively. Figure 3 depicts the projected locations of the mobiles and BSs where the game admits a unique or multiple NE. We observe that the game admits a unique NE in all cases except from a single location where both BSs are "equidistant" in terms of SIR levels.

We repeat the analysis above with 3 mobiles instead of 2 on the same network. While the game admits a NE in all of the 4913 sample points, there are 39 points with multiple NE corresponding to a percentage of 0.79%. Multiple NE solutions occur again at specific (symmetric) locations as shown in Figure 4. Figure 5, on the other hand, depicts an arbitrary subset of 4913 sample locations of mobiles for comparison purposes.

We now investigate the effect of system and user parameters on the NE solutions of the hybrid power control game. We first vary L by choosing its values from the set $\{64, 128, 192, 256\}$. For each value of L, 4913 location points are generated on a grid with sample distance of 0.25. It is observed that the game admits a NE in all instances. The percentage of samples with multiple NE, on the other hand, are shown in Figure 6. Next, we set $L = 128$ and vary the background noise at the BSs such that $\sigma_1^2, \sigma_2^2 \in \{0.1, 0.5, 1, 2\}$. Figure 7 depicts the percentage of multiple NE whereas there is again at least one NE solution in all cases. We finally investigate the effect of pricing parameters λ_1, λ_2 while setting $\sigma_1^2 = \sigma_2^2 = 0.1$ and $\lambda_3 = 0.1$. The results are shown in Figure 8. Analyzing the individual cases of the game admitting multiple NE, we conclude that a high percentage of multiple solutions is partly a result of one or more mobiles transmitting with zero NE power due to high prices. In other words, although these cases technically constitute multiple NE solutions, they do not play a significant role in practice. We refer to [3] for a discussion on the relationship between prices and (soft) admission control.

We observe that the simulations conducted using gridding techniques yield somewhat distorted results in terms of multiple NE solutions due to the inherent lattice structure which exhibits specific symmetry properties. Therefore, we repeat the analysis above with samples generated using random sampling methods. The locations of the mobiles are chosen randomly with uniform distribution on the support set defined by the boundaries of the network. We repeat the first three simulations with 1000 randomly generated location points. In accordance with the discussion in Section 4.1, 1000 sample points guarantee that our estimates on the percentage of the existence and uniqueness of NE solutions are accurate within $\epsilon = 6\%$ with a probability of at least $1 - \delta = 0.998$. In other words, $Probability\left(|\widehat{p}_N - p| < 0.06\right) \geq 0.998$, where $\widehat{p}_N$ and p denote respectively the estimated and real probabilities of the analyzed property. In all of these three simulations we observe that the game admits a unique NE with 100% estimated probability. Hence, the probability of having a unique NE solution in 94% of the possible configurations is at least 0.998. This clearly indicates that multiple NE solutions are obtained only at very specific symmetric configurations, which coincide with the lattice structure of the gridding techniques. Figure 9, on the other hand, shows the effect of varying pricing parameters $\lambda_1,\ \lambda_2$ where $\lambda_3 = 0.1$. We note an overall decrease in the number of instances of the problem with multiple NE solutions due to the random nature of samples.

Next, we consider a more realistic two-dimensional (2-D) wireless network model. The previous simulations are repeated on the 2-D network again using random sampling. We observe that almost all of the results are comparable with the ones on the 1-D network. Figure 10 depicts the effect of varying pricing parameters $\lambda_1,\ \lambda_2$ with $\lambda_3 = 0.1$. In a more realistic simulation, we investigate the existence and uniqueness of NE on a network of 3 BSs with 4 mobiles. The hybrid game admits a unique NE solution in all instances considered corresponding to a probability estimate of 100%. A subset of 1000 randomly generated location points of two of the mobiles and locations of the BSs are depicted in Figure 12.

Finally, we take the pricing function, P_i, in the user costs to be strictly convex, specifically quadratic, $P_i = \lambda_i p_i^2$, where λ_i is again a user-specific parameter. We simulate 3 mobiles on a 2-D wireless network with 2 BSs and study NE solutions for 100 randomly generated location points for varying pricing parameters $\lambda_1,\ \lambda_2$ where $\lambda_3 = 0.1$. In accordance with the discussion in Section 4.1, 100 sample points guarantee that our estimates on the percentage of the existence of NE solutions are accurate within $\epsilon = 15\%$ with a probability of at least $1 - \delta = 0.98$.

The results shown in Figure 11 are quite different from the ones in the linear pricing case. Instead of having a NE solution in 100% of the cases (and multiple NE in many of them) we observe that in the majority of the cases there is no pure NE solution. In addition, when it exists the NE solution is observed to be unique in all instances. This result is possibly due to the highly nonlinear nature of the pricing function.

5.2 System Dynamics and Convergence

We simulate a joint power update and BS assignment scheme for a 2-D wireless network consisting of 4 BSs and arbitrarily placed 20 mobiles. The system parameters are $L = 128$ and $\sigma_l^2 = 0.1 \; \forall l$ and a linear pricing scheme is chosen for this simulation. Locations of BSs and mobiles are shown in Figure 13.

In order to minimize its cost function (7), the i^{th} mobile chooses the BS that maximizes its SIR level and updates its transmission power level using the algorithm (6) such that

$$p_i^{(n+1)} = \frac{1}{\lambda_i} - \frac{1}{L\, h_{il}} \left(\sum_{j \neq i} h_{jl} p_j^{(n)} + \sigma_l^2) \right),$$

where n denotes the time, and BS choice of mobile i is $b_i = l$. In addition, through a projection operation it is ensured that $p_i^{(n)} > 0 \; \forall \, i, \, n$. Figures 14 and 15 depict the evolution of the power and SIR levels of the mobiles. We repeat the same simulation where the mobiles connect only to the nearest BS and update their power levels. The sums of the SIR levels achieved by the users in both cases are compared in Figure 16. Clearly, the additional freedom of BS choice, and hence, the hybrid power control game provides an improvement over "classical" noncooperative power control.

6 Conclusions

In this paper, we have studied a hybrid noncooperative game motivated by the practical problem of joint power control and BS assignment in CDMA wireless data networks, and have extended the noncooperative game theoretic approach in our earlier studies [3–5]. We have developed a hybrid power control game where mobiles are associated with a specific cost structure, and investigated the existence and uniqueness of pure NE solutions numerically using randomized algorithms. As part of our numerical analysis we have utilized both gridding techniques and random sampling. The results obtained indicate that the hybrid game admits a unique NE in most of the parameter configurations when a linear pricing scheme is used. We have also encountered multiple NE solutions in specific cases of high prices and symmetric locations of users with respect to the BSs. On the other hand, when a strictly convex pricing scheme is imposed on mobiles we have observed that there exists a pure NE solution only in a minority of the randomly generated configurations. In addition, we have not obtained any multiple NE solutions. Finally, we have simulated a dynamic BS assignment and power update scheme. Simulation results show that the power levels and SIR values of the users converge to their respective equilibrium points. Furthermore, the hybrid power control game has a distinct advantage over "classical" noncooperative power control algorithms in terms of aggregate SIR levels obtained by all users (for most parameter values and configurations). A possible extension of the results of this paper would involve analytical investigation of NE properties as well as convergence characteristics of the joint power update and BS assignment scheme.

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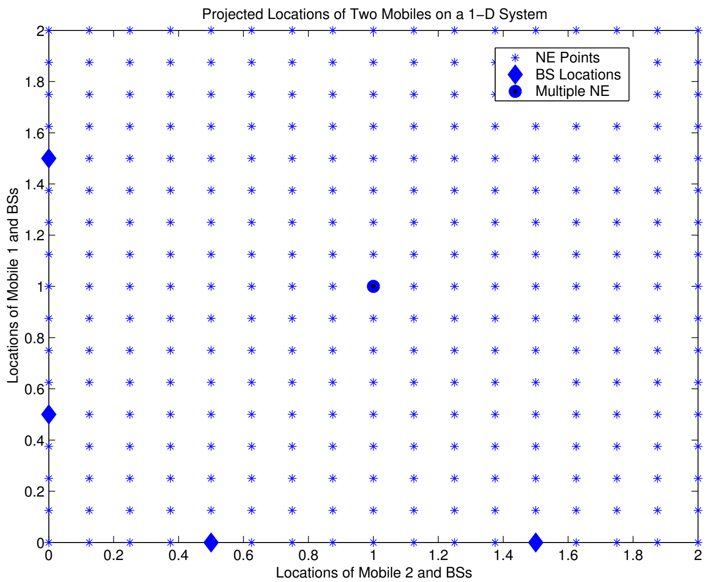

Figure 3: Projected locations of two mobiles on a 2BS 1-D network where the game admits a NE.

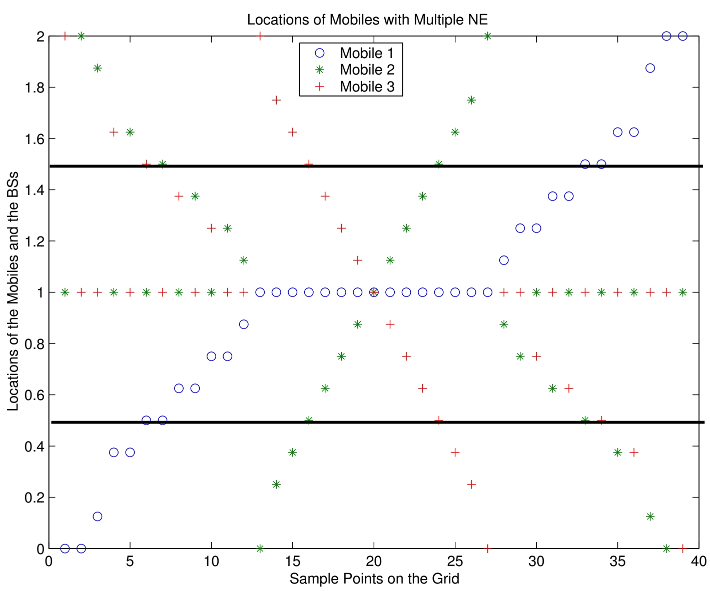

Figure 4: Locations of 3 mobiles on a 2BS 1-D network where the game admits multiple NE.

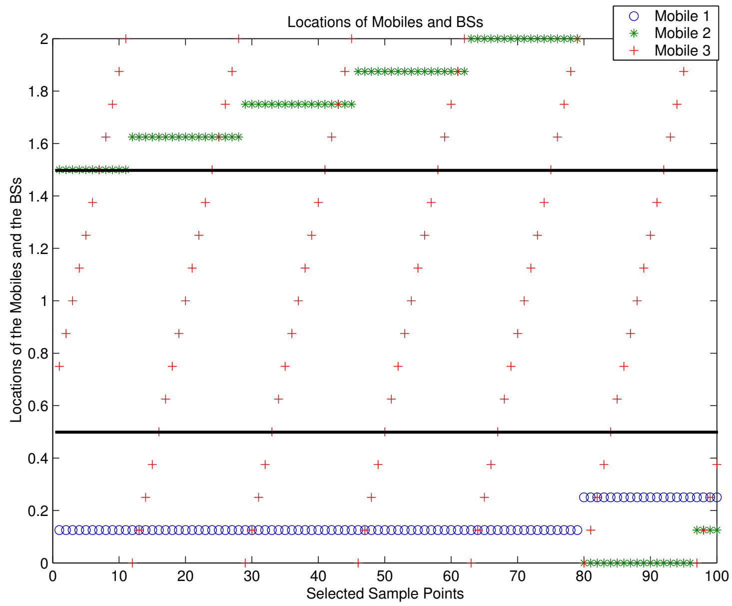

Figure 5: An arbitrary subset of the sample points depicting locations of 3 mobiles on a 2BS 1-D network.

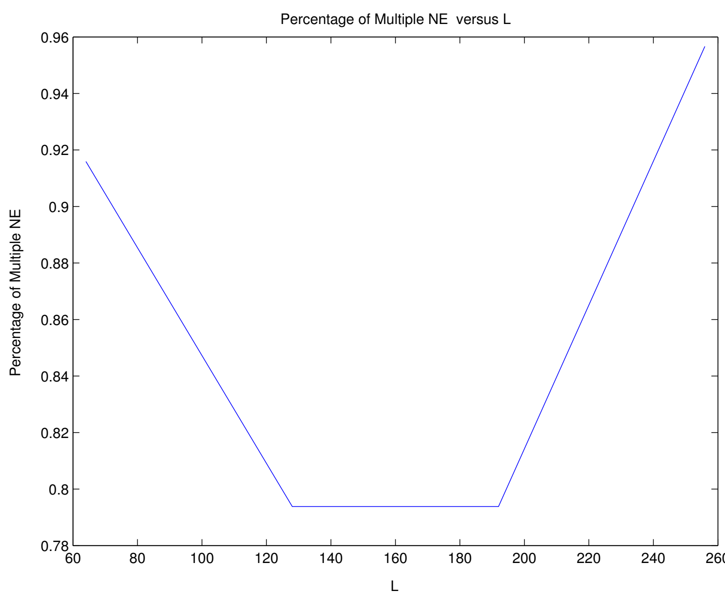

Figure 6: Percentage of mobile locations with multiple NE solutions for different values of L.

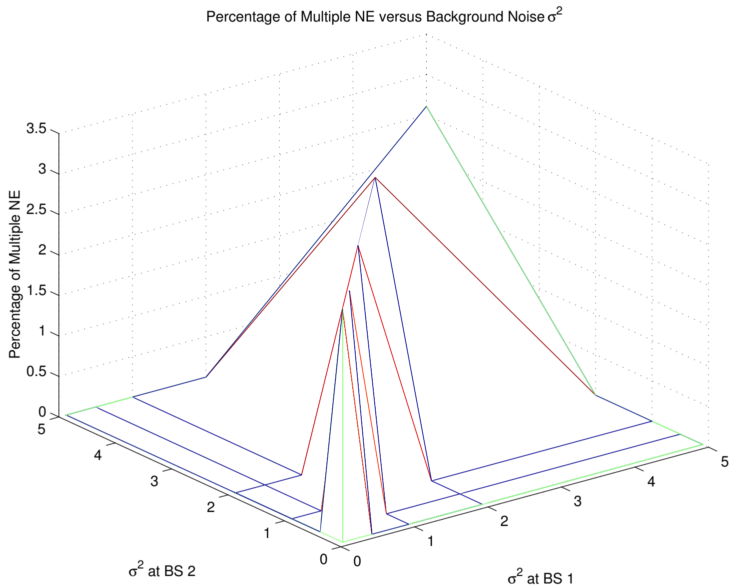

Figure 7: Percentage of mobile locations with multiple NE solutions for different values of σ_1^2 and σ_2^2.

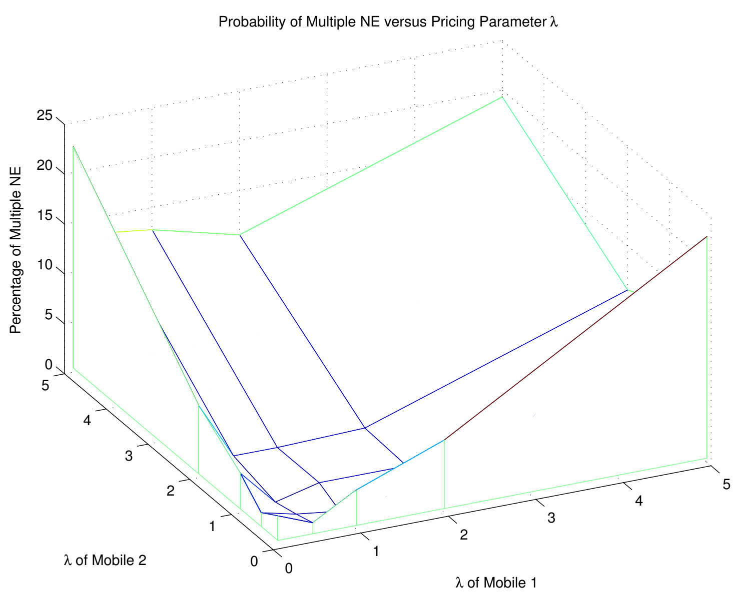

Figure 8: Percentage of mobile locations with multiple NE solutions for different values of λ_1 and λ_2.

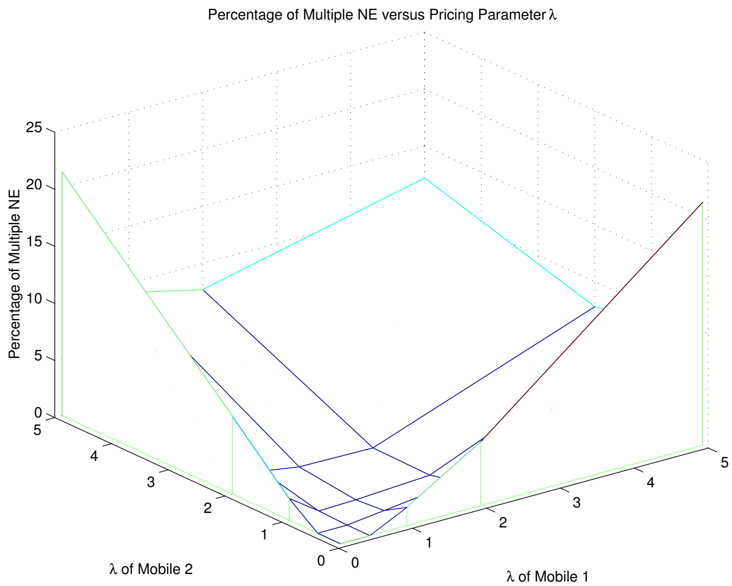

Figure 9: Percentage of mobile locations with multiple NE solutions for different values of λ_1 and λ_2.

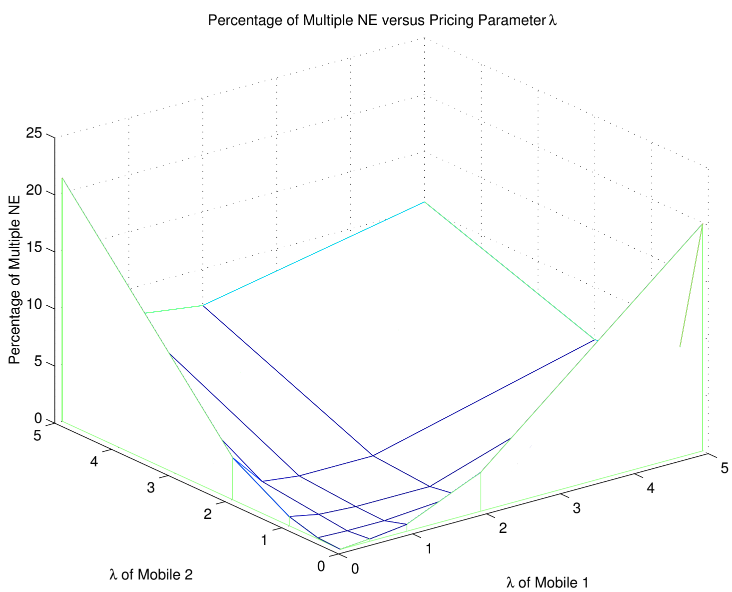

Figure 10: Percentage of mobile locations with multiple NE solutions for different values of λ_1 and λ_2 on a 2-D wireless network.

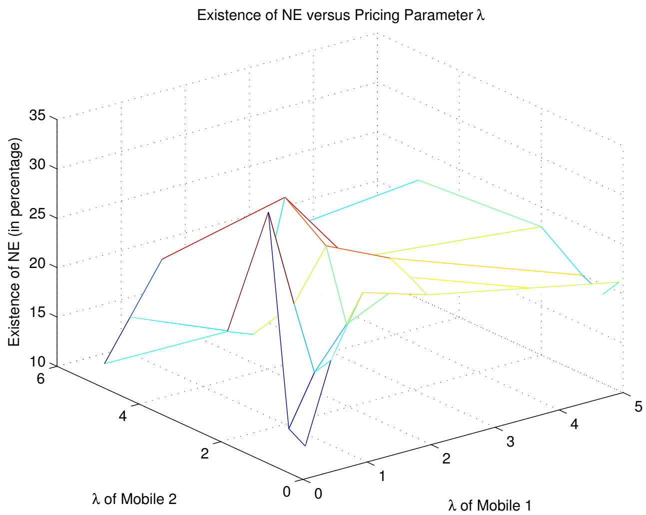

Figure 11: Percentage of mobile locations with NE solutions in the case of quadratic pricing function, $P_i = \lambda_i p_i^2$, for different values of λ_1 and λ_2 on a 2-D wireless network.

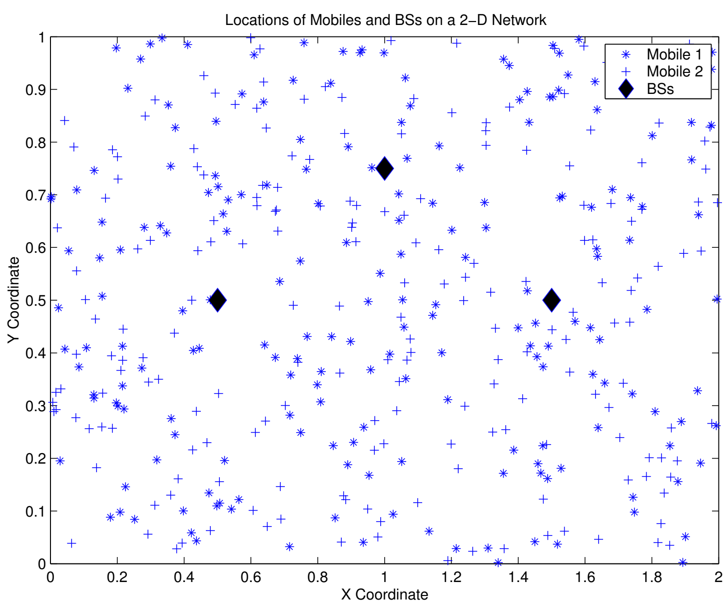

Figure 12: Randomly generated locations of two mobiles on a 2-D wireless network with 3 BSs.

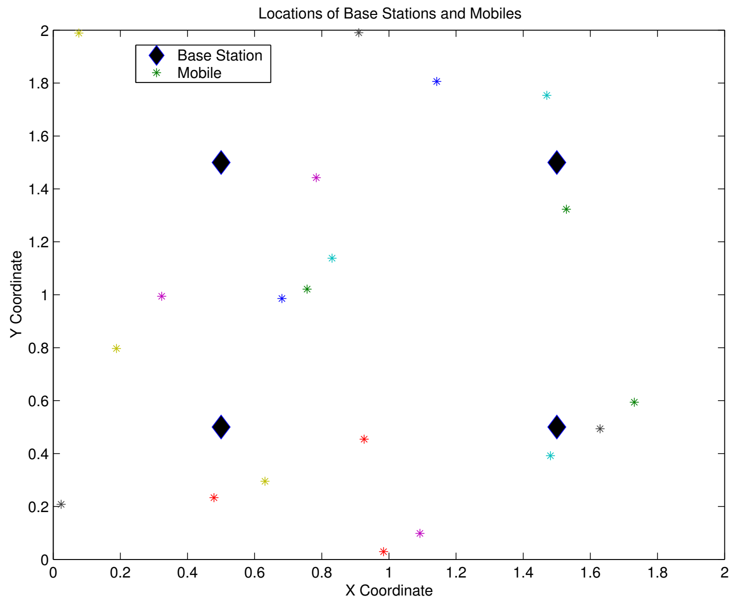

Figure 13: Locations of base stations and mobiles for dynamic simulations.

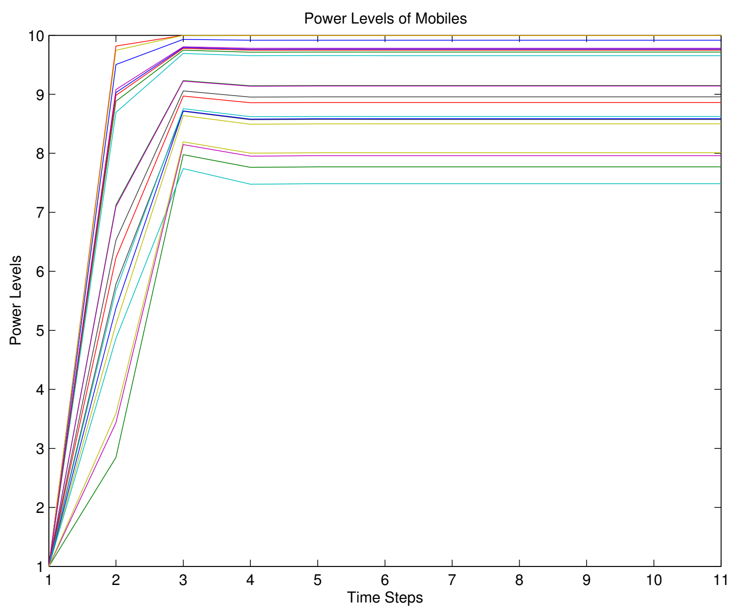

Figure 14: Power levels of mobiles with respect to time.

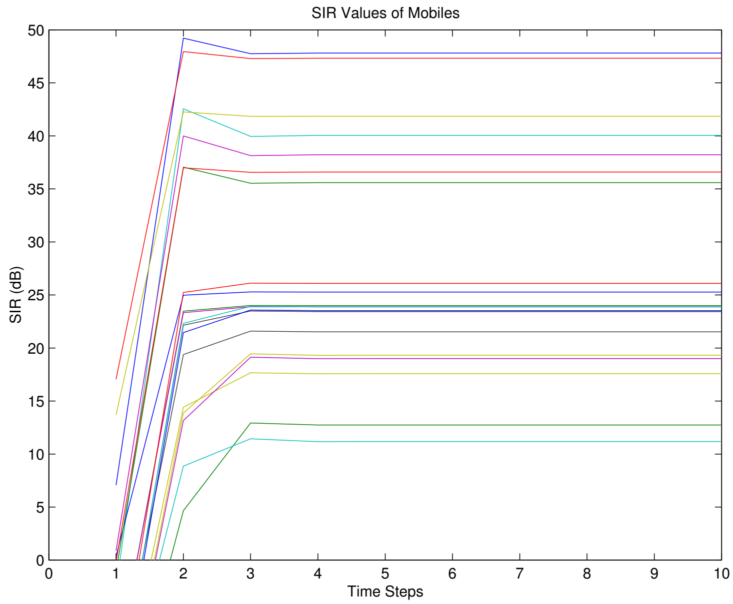

Figure 15: SIR values of mobiles (in dB) with respect to time.

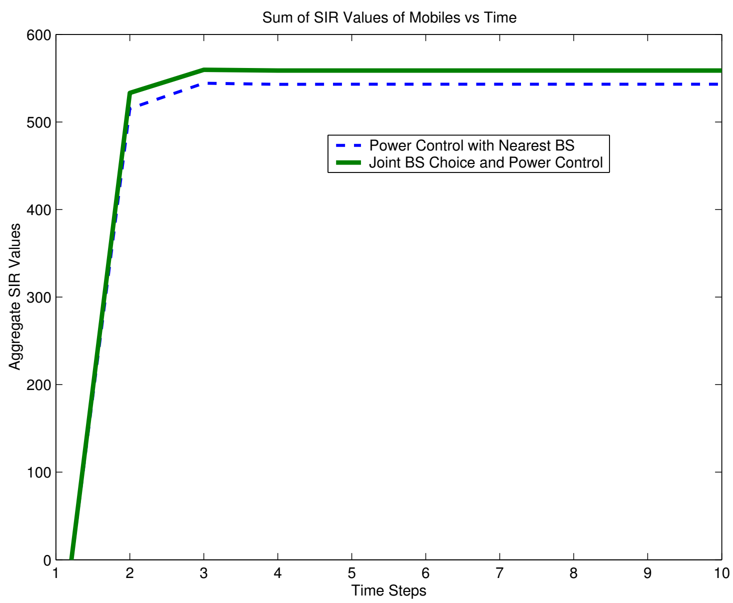

Figure 16: Sums of the SIR values of mobiles (in dB) with respect to time for power control with nearest BS choice and hybrid power control scheme.