

# Efficient and Strategy-Proof Mechanisms for General Concave User Utilities

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*Abstract*—This paper introduces a novel methodology for designing efficient and strategy-proof direct mechanisms for a class of problems, where the user types are represented by smooth, concave, and increasing utility functions. Such mechanisms facilitate distributed control and allocation of resources. Hence, they are applicable to diverse problems ranging from those in communication networks to energy management.

A three-step mechanism design process is presented for deriving the resource allocation and pricing functionals based on user bids in an auction setting. The properties of the resulting class of mechanisms are formally analysed using strategic (noncooperative) games. Although these mechanisms belong to the Groves class, they differ from the Vickrey-Clarke-Groves (VCG) mechanisms. The developed design process is illustrated with analytically tractable examples, which are motivated by network control problems and use scalar-parameterised logarithmic utility functions. It is shown that the resulting schemes are both efficient and truth-revealing (strategy proof) as expected.

## I. INTRODUCTION

### A. Background

Game theory, specifically **strategic (noncooperative) games**, study multi-person decision making by taking into account preferences of individual players (e.g users), who share and compete for limited resources in a system. They provide a suitable mathematical approach for formal analysis and design of distributed optimisation and control systems. Unsurprisingly, game theory enjoys widespread adoption by the engineering community. Game theoretic frameworks have been developed to address various problems such as rate control, interference management, and power control in wireless, wired, and optical networks [1], [2], [15].

Nash Equilibrium (NE) is an important solution concept in strategic games. It is defined as a fixed-point where no player of a strategic game has an incentive to deviate from. It follows directly from the action space and utilities of players through a fixed-point theorem [5] and its elegance lies partly in its simplicity. However, a Nash Equilibrium can be a very inefficient solution with respect to a given global objective. This issue known as *price of anarchy* or *efficiency loss* has been the subject of many investigations [8], [10], [11], [16], [18]. The problem gets more complicated in the case of multiple NE, where it is difficult to even define what it means to have an efficient outcome. Therefore, the focus here is on a class of strategic games which admit a unique NE solution.

It is clearly desirable to **design mechanisms**, which can be formally analysed using game theory, such that their outcome is efficient, or the corresponding equilibrium of the associated game is optimal with respect to a given global objective [3], [4]. A *mechanism designer* aims to develop and implement such an optimal mechanism **without having prior knowledge** (possibly except from aggregate statistics) **on participating users' preferences, which are captured here by smooth, concave, and increasing utility functions**. The users in the system, modelled as players of the corresponding game, are free to behave according their to own private and selfish incentives which may contradict the global objectives. *The problem in this case turns out to be the information exchange between the mechanism (designer) and users*. If the **users** have sufficient knowledge on the system and its operation, they **can try to mislead the designer or manipulate the mechanism** such that the outcome is to their individual benefit in expense of others. This result is clearly not desirable from a mechanism design perspective. Therefore, a desirable mechanism has to be not only **efficient** but also **truth revealing** or *strategy proof*, i.e. the users have no incentive to mislead the system [6], [11], [12], [14].

The *efficiency* criterion means that the equilibrium of the game modelling the mechanism coincides with the maximum of a given global objective function. The commonly used “social welfare” of users, which is the (weighted) sum of user utilities, is chosen here as the global objective function. *It is assumed that the designer does not have even statistical or aggregate knowledge of user utilities (preferences or types)*. Consequently, mechanisms captured by Bayesian games are not investigated here. The mechanisms developed here are referred to as *truth revealing* or strategy proof, if and only if the corresponding game admits a dominant strategy equilibrium (DSE), which uniquely reveals the user types (preferences). A game admits a DSE, if the individual players choose an action regardless of the actions of others.

This paper presents a novel methodology for designing efficient and strategy-proof direct mechanisms for a specific class of problems. The resulting auction schemes are modelled and analysed as strategic games. The goal is then to ensure that the game admits a unique DSE and it coincides with the maximum of the sum of user utilities. Hence, the mechanism is provably efficient and truth revealing (strategy proof). This result is achieved by carefully choosing the relevant (resource) allocation and pricing functionals based on a three-step constructive design process. The design process is illustrated with example formulations motivated by network control problems and based on scalar-parameterised

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logarithmic user utilities. It is shown that the resulting mechanisms are indeed both efficient and truth-revealing (strategy proof).

## B. Contributions

The mechanism design approach introduced in this paper is novel and constructive. The mechanisms obtained differ from the well-known Vickrey-Clarke-Grove (VCG) mechanisms. Furthermore, the constructive nature of the design process allows derivation of a variety of mechanisms for different problem formulations. Although the class of mechanisms designed differ from VCG, they are still efficient in the sense of maximising sum of player utility functions and are strategy-proof, i.e. the corresponding strategic game admits a dominant strategy equilibrium which reveals the true user preferences. These properties can also be verified independently by checking whether the specific mechanisms obtained belong to the general class of Groves mechanisms, of which VCG is another special case [17].

The rest of the paper is organised as follows. The next section introduces the underlying model. Section III presents the main results in the context of auction-based mechanisms. The paper concludes with remarks of Section IV.

## II. MODEL

Consider an auction mechanism where a *designer*  $\mathcal{D}$  influences a finite set,  $\mathcal{A}$  of users who have private preferences and compete for limited resources through their bids. It is possible to represent these preferences using *utility functions*, and interpret them as user types. This paper focuses on sharing of an additive limited resource.<sup>1</sup> Each user receives and pays for a  $0 \leq q_i \leq C$  share of the total resource  $C$  as a result of an auction mechanism, such that

$$\sum_i q_i \leq C.$$

The designer tries to achieve a global objective such as welfare maximisation by making the users reveal their true utilities. For this purpose, the designer imposes certain rules and prices to the users agreeing to participate in the mechanism. However, the designer cannot dictate user actions or modify their private utility functions.

In order to formally analyze the considered auction mechanism, define an  $N$ -player strategic game,  $\mathcal{G}$ , where each user  $i \in \mathcal{A}$  makes a **bid**

$$x_i(q_i) \in \mathcal{C}^2[0, C],$$

which is defined as a continuous, strictly concave, and twice-differentiable function of the users share of the resource,  $q_i$ , on the interval  $[0, C]$ . The function  $x_i$  represents the **declared preference**, utility, or willingness-to-pay of user  $i$  for the resource  $q_i$ .

<sup>1</sup>Note that, the users are greedy enough such that the demand for the resource is more than its availability. Otherwise, there would not be a resource allocation problem since the designer would have allocated each user simply its desired amount of resource without affecting other users.

The **real utility** of the  $i^{\text{th}}$  user for the received resource  $q_i$  is captured by the utility function

$$U_i(q_i) : \mathbb{R} \rightarrow \mathbb{R},$$

which is also assumed to be continuous, twice-differentiable, and concave on the interval  $[0, C]$ . **The focus here is on direct and truth-revealing mechanisms, where in the ideal case the users' bids are their real utility functions.**

The designer imposes a pricing signal on the bids of users, which is formulated by adding it as a cost term to utility resulting in a *quasilinear* setting. Hence, the user  $i$  has the quasilinear cost functional

$$J_i(x) = c_i(x) - U_i(Q_i(x)), \quad (1)$$

where  $c_i$  and  $Q_i$  are the pricing and allocation functionals mapping  $\mathcal{C}^2[0, C] \rightarrow \mathbb{R}$ , respectively. Consequently, the user solves the individual optimisation problem

$$\min_{x_i} J_i(x). \quad (2)$$

It is important to note that we assume here *price anticipating* users, who take into account the effect of their actions on prices  $c_i(x)$  and act accordingly. This is in contrast with *price taking* users who ignore it at least partially, e.g. due to lack of information.

The **Nash equilibrium** (NE) is a widely-accepted and useful solution concept in strategic games, where no player has an incentive to deviate from it while others play according to their NE strategies. The NE  $x^*$  of the game  $\mathcal{G}$  is formally defined as

$$x_i^* := \arg \min_{x_i} J_i(x_i, x_{-i}^*),$$

where  $x_{-i}^* = [x_1^*, \dots, x_{i-1}^*, x_{i+1}^*, \dots, x_N^*]$ . The NE is at the same time the intersection point of players' best responses obtained by solving (2) individually.

A stronger concept is **Dominant Strategy Equilibrium** (DSE), which is defined as

$$x_i^D := \arg \min_{x_i} J_i(x_i, x_{-i}), \quad \forall x_{-i} \forall i,$$

i.e. the players choose the dominant strategy regardless of the actions of others.

The **designer objective**, e.g. maximisation of aggregate user utilities or social welfare, can be formulated using a smooth objective function  $V(x, U_i(x))$ , where  $U_i(x)$ ,  $i = 1, \dots, N$  are user-specific pricing terms and user utilities, respectively. Thus, the objective function  $V$  characterises the desirability of an outcome  $x$  from the designers perspective. Interestingly, the designer knows user bids  $x$  as well as the mechanism ( $c$  and  $Q$ ), however, has no access to true user utilities  $U$ .

The following definitions describe various properties of a mechanism and its corresponding game counterpart:

**Definition II.1** (Efficiency). *A mechanism is said to be efficient if its outcome, i.e. the NE or DSE of the corresponding strategic game,  $x^*$ , satisfies*

$$x^* = \arg \max_x V(x, U_i(x)),$$

where  $V$  is the objective function of the designer.

**Definition II.2** (Strategy-proof). A mechanism is said to be strategy-proof, if and only if, the corresponding game admits a DSE that reveals the true user types (preferences).

**Definition II.3** (Revelation). In a strategy-proof mechanism, each rational user acts according to own true utility or reveals own true type regardless of the actions of others, i.e. does not try to mislead the designer.

Note that these definitions are consistent with the properties of quasilinear mechanisms as discussed in [17].

### III. MECHANISM DESIGN

In **auction-based mechanisms**, the designer uses an allocation rule in addition to impose a cost on user actions. Based on this rule, the designer *explicitly allocates* the users their share of resources as a result of their bids. Specifically, the designer  $\mathcal{D}$  imposes on a user  $i \in \mathcal{A}$  (possibly a user-specific)

- resource allocation rule,  $Q_i(x)$ , s.t.  $q_i = Q_i(x)$ ,
- per-unit resource prices,  $P_i(x)$ ,

where  $x$  denotes the vector of user bids.

As presented in Section II, each user  $i$  aims to minimise its own cost  $J_i(Q_i(x), P_i(x))$ , as in (1), while the designer tries to maximise a global objective  $V$ . The **interaction between the designer and users**, depicted in Figure 1, is through a single-step **bidding/allocation process** in the auction-based mechanisms defined. Since the users cannot obtain the resource  $q$  directly, they make a bid  $x$  for their own share. Since this is a direct mechanism, these bids are in the ideal case the actual utilities of users.

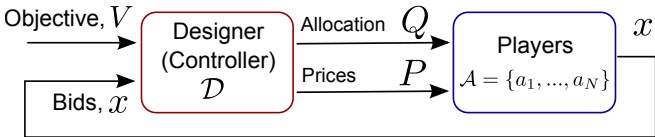


Fig. 1. An auction-based mechanism, where the designer  $\mathcal{D}$  imposes a resource allocation rule as well as prices on users (players)  $\mathcal{A}$  with the purpose of satisfying a global objective  $V$ .

The main steps for designing an efficient and strategy-proof auction mechanism are:

- 1) (*efficiency*) Define and solve user and designer optimisation problems in terms of resources,  $q$ , i.e. identify the equilibrium and globally optimal point.
- 2) (*efficiency*) Align the user and designer problems using the Lagrange multiplier(s) of the limited resource, i.e. move the equilibrium to the optimal point.
- 3) (*strategy-proof*) Devise the allocation rule (functional),  $q$ , based on the problem alignment and then choose a pricing functional that ensures a truth-revealing dominant strategy equilibrium (DSE).

### User Problem

The  $i^{\text{th}}$  users individual cost functional  $J_i(x)$  in terms of all user bids  $x$  is defined as

$$J_i(x) = c_i(x) - U_i(Q_i(x)).$$

Note that  $J_i$  is a functional, which we assume to be Frechet differentiable at the points of interests. Before proceeding further, it is appropriate to define the Frechet differential and derivative for completeness. Let  $\mathcal{X} = (\mathcal{C}^2[0, C])^n$  be the space of continuous, concave, and twice differentiable  $n$ -vector functions on  $[0, C]$  and define the functional  $T : \mathcal{X} \rightarrow \mathbb{R}$ . If for fixed  $x \in \mathcal{X}$  and each  $h \in \mathcal{X}$ , there exists  $\delta T(x; h)$ , which is linear and continuous with respect to  $h$  such that

$$\lim_{\|h\| \rightarrow 0} \frac{\|T(x+h) - T(x) - \delta T(x; h)\|}{\|h\|} = 0,$$

then  $\delta T(x; h)$  is said to be the Frechet differential of  $T$  at  $x$  with increment  $h$ . Furthermore,  $\delta T(x; h) = T'(x)h$  and  $T'(x)$  is called the Frechet derivative [13].

Taking the Frechet differential [13] of the user cost functional with respect to own bid  $x_i$  results in

$$\delta J_i(x_i, x_{-i}; h) = \delta c_i(x_i, x_{-i}; h) - \frac{\partial U_i}{\partial Q_i} \delta Q_i(x_i, x_{-i}; h).$$

Let  $P_i$  and  $Q'_i$  be the Frechet derivatives of  $c_i$  and  $Q_i$ , respectively. Then, the Frechet differential can be written as

$$\delta J_i(x_i, x_{-i}; h) = P_i(x)h - \frac{\partial U_i}{\partial Q_i} Q'_i h. \quad (3)$$

Given  $x_{-i}$ , the best response of user  $i$ ,  $x_i^*$ , consequently satisfies

$$\delta J_i(x_i, x_{-i}; x_i - x_i^*) \geq 0 \quad \forall x_i \in \mathcal{C}[0, C].$$

Assuming that the functional  $J_i$  is convex, then the local solution to the user problem constitutes concurrently the global (non-boundary) solution.

### Designer Problem

The designer  $\mathcal{D}$  aims to maximise the sum of utilities of users. Clearly the case where the optimal solution is obtained at  $\sum_i q_i = C$  is of interest. Otherwise, users can solve their own problems independently without a need for a mechanism, i.e. the resource is abundant enough to satisfy the needs of all users.

In the resource-limited case, the designer  $\mathcal{D}$  solves the constrained optimisation problem

$$\max_q V(q) \Leftrightarrow \max_q \sum_i U_i(q_i) \text{ such that } \sum_i q_i = C, \quad (4)$$

in terms of the allocated resources  $q = Q(x)$ .

The associated Lagrangian function is

$$L(q) = \sum_i U_i(q_i) + \lambda \left( C - \sum_i q_i \right),$$

where  $\lambda > 0$  is a scalar Lagrange multiplier. The derivatives

of the Lagrangian lead to

$$\frac{\partial L}{\partial q_i} = 0 \Rightarrow U_i'(q_i) = \lambda^*, \forall i \in \mathcal{A}, \quad (5)$$

and the efficiency constraint

$$\frac{\partial L}{\partial \lambda} = 0 \Rightarrow \sum_i q_i = C. \quad (6)$$

### Efficiency

Next, the designer problem is solved to obtain the allocation functional (rule)  $Q(x)$  such that the equilibrium point overlaps with the optimal solution.

Solving the set of equations (5) and (6) from the designer problem for  $q$  yields the allocation functional

$$Q(x, \lambda^*(x)) = Q(x),$$

where  $\lambda^*$  is defined in (5). This allocation rule *defined in terms of user bids* provides a Pareto-optimal solution, if the user bids are made according their true utilities,  $U_i$ .

### Strategy-proofness

Finally, the pricing functional  $P_i(x)$  is designed in such a way that the Pareto solution coincides with the DSE of the game and users are forced to reveal their true preferences. Define the pricing functional as

$$P_i(x_i, x_{-i}) = x_i Q_i'(x),$$

where  $Q_i'$  is the Frechet derivative of  $Q_i$  with respect to  $x_i$ . Then, substituting  $P_i$  in (3) leads to

$$\delta J_i = \left( x_i - \frac{\partial U_i}{\partial Q_i} \right) Q_i' h.$$

Taking a second Frechet derivative yields

$$\delta^2 J_i = Q_i' h^2 - \left( \frac{\partial^2 U_i}{\partial Q_i^2} \right) (Q_i' h)^2 + \left( x_i - \frac{\partial U_i}{\partial Q_i} \right) Q_i'' h^2.$$

If  $Q_i' \geq 0$ , then the marginal utility of user  $i$ ,  $x_i^* = \partial U_i / \partial q_i$ , is the optimal bid or action that minimises users cost since  $\delta J_i = 0$  and  $\delta^2 J_i > 0$  regardless of the bids of all other users,  $x_{-i}$ . Therefore,  $x_i^* = \partial U_i / \partial q_i \forall i$  is the DSE of the strategic game, which reveals the true user utilities (preferences). Thus, the mechanism with the pricing functional

$$P_i(x_i, x_{-i}) = x_i Q_i'(x),$$

and allocation functional

$$Q_i(x) = x_i^{-1}(\lambda^*), \text{ where } \sum_i x_i^{-1}(\lambda^*) = C,$$

is strategy-proof.

Note that, the monotonicity condition imposed,  $Q_i' \geq 0$ , has a very reasonable interpretation. It means that the allocation to the users will be non-decreasing in their bids, i.e. the more a user bids, the more (or at least as much as before) will be her or his allocation.

### Example 1

Consider a game where the user utility functions are symmetric except from a scalar parameter  $\theta$ ,

$$U_i(q_i) = \theta_i \log(q_i).$$

This family of user utilities are often used to model user preferences in the networking literature, e.g. in flow or congestion control problems. Hence, the optimal user bid is the function

$$x_i(q_i) = \frac{\hat{\theta}_i}{q_i}.$$

From each bid, the designer learns the user parameters  $\hat{\theta}$ .

Solving designer problem leads to

$$Q_i^* = \frac{\theta_i}{\sum_i \theta_i} C \text{ and } \lambda^* = \frac{\sum_i \theta_i}{C}.$$

Hence, the allocation functional simplifies to a function of  $\hat{\theta}$  and is

$$Q_i(\hat{\theta}) = \frac{\hat{\theta}_i}{\sum_i \hat{\theta}_i} C.$$

Likewise, the pricing functional becomes a function of  $\hat{\theta}$  and is given by

$$P_i(\hat{\theta}) = \frac{\sum_{j \neq i} \hat{\theta}_j}{\sum_i \hat{\theta}_i},$$

or alternatively

$$P_i = 1 - \frac{Q_i}{C}.$$

The user prices are then

$$c_i(\hat{\theta}) = \left( \sum_{j \neq i} \hat{\theta}_j \right) \log \left( \sum_i \hat{\theta}_i \right).$$

Thus,  $x_i^* = \theta_i / q_i, \forall i$  is the DSE of the corresponding game, and hence, the mechanism is strategy-proof by Definition II.2. Furthermore, the mechanism belongs to the Groves class.

### Example 2

The analysis above is repeated for a slightly different family of utility functions

$$U_i(q_i) = \theta_i \log(q_i + 1).$$

Here, it is assumed that  $\theta_i$  is sufficiently large for each user such that  $q_i > 0 \forall i$ . Then, solving the designer problem yields

$$q_i^* = \frac{\theta_i}{\sum_i \theta_i} (C + N) - 1 \text{ and } \lambda^* = \frac{\sum_i \theta_i}{C + N} = \frac{\theta_i}{q_i^* + 1}.$$

Again, the user bids reveal their utility parameter  $\hat{\theta}$ . Hence, the allocation rule is defined as the function

$$Q_i^*(\hat{\theta}) = \frac{\hat{\theta}_i}{\sum_i \hat{\theta}_i} (C + N) - 1.$$

The rest of the analysis is similar to that in Example 1

above, which results in the same pricing function as before

$$P_i(x) = \frac{\sum_{j \neq i} \hat{\theta}_j}{\sum_i \hat{\theta}_i}, \quad P_i = 1 - \frac{Q_i}{C}.$$

Thus, the game admits a DSE and concurrently the mechanism is strategy-proof.

### Example 3

Another common user utility function is

$$U_i(q) = \theta_i \log(\gamma_i(q)),$$

where

$$\gamma_i(q) = \frac{q_i}{\sum_{j \neq i} q_j + \sigma} \quad (7)$$

denotes the signal-to-interference and noise ratio (SINR) and  $\sigma > 0$  is the noise variance. Such utility functions are often encountered in transmission power control problems in wireless networks [7], [9]. In this case,  $q_i$  represents the assigned (received or aimed) user power level and  $\sum_i q_i \leq C$  is the received sum power constraint for minimising total interference in the overall system. The objective is to maximise the aggregate utility of users in terms of SINR.

The resulting designer problem

$$\max_q \sum_i \theta_i \log(\gamma_i(q)) \text{ such that } \sum_i q_i \leq C, \quad q_i \geq 0 \quad \forall i$$

is non-convex but can be convexified using a nonlinear (exponential) transform such that it admits a unique boundary solution. Then, using the fact that  $\sum_i q_i = C$ , the problem can be written in terms of SINR

$$\max_{\gamma} \sum_i \theta_i \log(\gamma_i) \text{ such that } \sum_i \frac{\gamma_i}{\gamma_i + 1} = \frac{C}{C + \sigma}.$$

Note that, the vector  $\theta$  is now handled as an independent variable.

The respective Lagrangian

$$L = \sum_i \theta_i \log(\gamma_i) + \lambda \left( \frac{C}{C + \sigma} - \sum_i \frac{\gamma_i}{\gamma_i + 1} \right)$$

leads to

$$\lambda = \frac{(\gamma_i + 1)^2}{\gamma_i} \theta_i \quad \forall i \quad \text{and} \quad \sum_i \frac{\gamma_i}{\gamma_i + 1} = \frac{C}{C + \sigma}. \quad (8)$$

The optimal user bids

$$x_i = \frac{dU_i}{dq_i} = \frac{\hat{\theta}_i}{q_i}$$

provide the declared preference parameters,  $\hat{\theta}$ , which can be used to solve  $\lambda^*$  and  $\gamma^*$  in (8). Thus, the allocation function

$$Q_i(\hat{\theta}, \lambda^*(\hat{\theta}), \gamma^*(\hat{\theta})) = Q_i(\hat{\theta})$$

is obtained, but can be computed only numerically. The pricing function is then

$$P_i(\hat{\theta}) = \frac{\hat{\theta}_i}{Q_i} \frac{\partial Q_i}{\partial \hat{\theta}_i}.$$

## IV. CONCLUSIONS

A mechanism design approach is presented for deriving a class of efficient and strategy-proof auction mechanisms, where user preferences are captured by a class of smooth, concave, and increasing utility functions. A three-step design process is illustrated with multiple example user utilities that are commonly used in the network control literature. The mechanisms obtained as a result of this design process differ from VCG, yet can be shown to belong to the general class of Groves mechanisms in certain cases [17].

The results obtained extend the earlier ones with scalar-parameterised user utilities, where the shape of the user utility functions were assumed to be known by the mechanism designer. Removing this assumption significantly improves the applicability of this mechanism design framework to problems in various fields such as communication networks, energy management, and network security.

Future research directions include analysis of information exchange between mechanism designer and users, as well as extensions of the results to pricing (Pigovian) mechanisms and multiple constraints.

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