CDMA Uplink Power Control as a Noncooperative Game [1]

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Abstract

We present a game-theoretic treatment of distributed power control in CDMA wireless systems. We make use of the conceptual framework of noncooperative game theory to obtain a distributed and market-based control mechanism. We address not only the power control problem, but also pricing and allocation of a single resource among several users. A cost function is introduced as the difference between pricing and utility functions, and the existence of a unique Nash equilibrium is established. Furthermore, two update algorithms, namely parallel update and random update, are shown to be globally stable under specific conditions. Convergence properties and robustness of each algorithm are also studied through extensive simulations.

1 Introduction

In wireless communication systems, mobile users respond to the time-varying nature of the channel, described using short-term and long-term fading phenomena [1], by regulating their transmitter powers. Specifically, in a code division multiple access (CDMA) system, where signals of other users can be modeled as interfering noise signals, the major goal of this regulation is to achieve a certain signal to interference (SIR) ratio. Hence, there are two major reasons for a user to exercise power control: the first is to conserve battery energy at the mobile, and the second is to minimize the effect of interference.

A specific scheme for distributed power control introduced in [2] relies on each user updating its power based on the total received power at the base station. It has been shown in [2] that the resulting distributed power control algorithm converges under a wide variety of interference models. Another distributed power control scheme has been introduced in [3], which is adaptive and uses local measurements of the mean and the variance of the interference. The authors have shown that this algorithm is convergent under a certain condition. Game theory provides a natural framework for developing pricing mechanisms of direct relevance to the power control problem in wireless networks. In such networks, the users behave noncooperatively, i.e., each user attempts to minimize its own cost function (or maximize its utility function) in response to the actions of the other users. This makes the use of noncooperative game theory [4] for uplink power control most appropriate, with the relevant solution concept being the noncooperative Nash equilibrium (NE). In this approach, a noncooperative network game is defined where each user attempts to minimize a specific cost function by adjusting his transmission power, with the remaining users' power levels fixed. The main advantage of this approach is that it not only leads to distributed control as in [2], but also naturally leads to pricing schemes, as we will see in this paper.

Possible utility functions in a game theoretical framework, and their properties for both voice and data sources have been investigated in [5], which formulates a class of utility functions that also account for error correction, and shows the existence of a unique NE. One interesting feature of this framework is that it provides utility functions for wireless data transmission, where power control directly affects the capacity of mobiles' data transmission rates. Reference [5] also proposes a linear pricing scheme in order to achieve a Pareto improvement in the utilities of mobiles. In an earlier study [6], Nash equilibria achieved under a pricing scheme have been characterized by using supermodularity. It has been shown that a noncooperative power control game with a pricing scheme is superior to one without pricing. One deficiency of this game setup, however, is that it does not guarantee social optimality for the equilibrium points.

In the model we adopt in this paper, we use a cost function defined as the difference between a linear pricing scheme proportional to transmitted power, and a logarithmic, strictly concave utility function based on SIR of the mobile. We then prove the existence and uniqueness of a NE. As in [2], one way of extending the model is to include certain SIR constraints. As an alternative, we suggest a pricing strategy to meet the given constraints, and analyze the relation between price and SIR. We study different pricing strategies, and obtain a sufficient condition for convergence of two algorithms, parallel update (PUA) and random update (RUA), to

[1] Research supported in part by grants NSF ANI 98-13710, NSF INT 98-04950, NSF CCR 00-85917, AFOSR MURI AF DC 5-36128, and ARMY OSP 35352-6086.

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the unique NE.

The next section describes the model and the cost function. In Section 3, we prove the existence and uniqueness of NE. We present update algorithms for mobiles in Section 4, whereas Section 5 introduces different pricing strategies at the base station. Simulation results are given in Section 6, which is followed by the concluding remarks of Section 7.

2 The Model and Cost Function

We describe here the model adopted in this paper for a single cell CDMA system with up to M users. For the i^{th} user, we define the cost function J_i as the difference between the utility function of the user and its pricing function, $J_i = P_i - U_i$. The utility function, U_i, is chosen as a logarithmic function of the i^{th} user's SIR, which we denote by γ_i. This utility function can be interpreted as being proportional to the Shannon capacity [1] for user i, if we make the simplifying assumption that the noise plus the interference of all other users constitute an independent Gaussian noise. This means that this part of the utility is simply linear in the throughput that can be achieved (or approached) by user i using an appropriate coding, as a function of its transmission power. This logarithmic function is further weighted by a user-specific utility parameter, $u_i > 0$, to capture the user's level of "desire" for SIR.

The pricing function defines the instantaneous "price" a user pays for using a specific amount of power that causes interference in the system. It is a linear function of p_i, the power level of the user. Accordingly, the cost function of the i^{th} user is defined as

$$J_i(p_i, p_{-i}) = \lambda_i \, p_i - u_i \ln(1+\gamma_i) \ , \quad p_i > 0 \; \forall i, \qquad (2.1)$$

where p_{-i} is the vector of power levels of all users except the i^{th}, and γ_i is given by

$$\gamma_i = L \frac{h_i p_i}{\sum_{j \neq i} h_j p_j + \sigma^2} \ . \qquad (2.2)$$

Here, $L = W/R$ is the spreading gain of the CDMA system, where W is the chip rate and R is the total rate; we assume throughout that $L > 1$. The parameter h_j, $0 < h_j < 1$, is the channel gain from user j to the base station in the cell, and $\sigma^2 > 0$ is the interference. For notational convenience, let us denote the i^{th} user's power level received at the base station as $y_i := h_i p_i$, introduce the quantity $\bar{y}_{-i} := \sum_{j \neq i} y_j$, and further define a user specific parameter (a_i) for the i^{th} user as $a_i := (u_i h_i / \lambda_i) - (\sigma^2 / L)$.

3 Existence and Uniqueness of Nash Equilibrium

The i^{th} user's optimization problem is to minimize its cost (2.1), given the sum of powers of other users as received at the base station, $\bar{y}_{-i}$, and noise, under the nonnegativity constraint, $p_i \geq 0$. The resulting unique reaction function, Φ_i, of the i^{th} user, is

$$p_i = \Phi_i(\bar{y}_{-i}, a_i) = \begin{cases} \frac{1}{h_i}[a_i - \frac{1}{L}\bar{y}_{-i}], & \text{if } \bar{y}_{-i} \leq L a_i \\ 0 \ , & \text{else} \end{cases} \qquad (3.1)$$

This reaction function depends not only on the user-specific parameters, like u_i, λ_i, and h_i, but also on the network parameter, L, and $\bar{y}_{-i}$. Similar to the transmission control protocol (TCP) in the Internet [7], there is an inherent feedback mechanism here, built into the reaction function of the user. Here the total received power at the base station provides the user with information about the "demand" in the network, which is comparable to congestion in case of the TCP. However, one major difference is that here the reaction function itself takes the place of the window based algorithms in the TCP.

We deduce the following conditions from (3.1) in order for the mobile to be "active," or $p_i > 0$. The first is the positivity of a_i; this can be equivalently interpreted as a lower bound on the utility parameter u_i. The second condition is $\bar{y}_{-i} \leq L a_i$, which follows directly from (3.1). Any equilibrium solution will have to constitute a solution to (3.1). If all M users have positive power levels at equilibrium, then the set of equations to solve is

$$L y_i + \bar{y}_{-i} = L a_i \ , \ i = 1, \ldots, M \ , \qquad (3.2)$$

whereas if some of the users have zero power levels at equilibrium, then an appropriate modification will have to be made to (3.2). The following theorem now captures this and provides the complete NE solution.

Theorem 3.1 *In the power game just defined (with M users), let the indexing be done such that $a_i < a_j \Rightarrow i > j$, with the ordering picked arbitrarily if $a_i = a_j$. Let $M^* \leq M$ be the largest integer $\tilde{M}$ for which the following condition is satisfied:*

$$a_{\tilde{M}} > \frac{1}{(L + \tilde{M} - 1)} \sum_{i=1}^{\tilde{M}} a_i \ . \qquad (3.3)$$

Then, the power game admits a unique NE, which has the property that users $M^*+1, \ldots, M$ have zero power levels, $p_j^* = 0 \; j \geq M^*+1$. The equilibrium power levels of the first M^* users are obtained uniquely from (3.2) with M replaced by M^*, and are given by, *for* $i \in \mathbf{M}^* := \{1, 2, \ldots, M^*\}$,

$$p_i^* = \frac{1}{h_i}\left\{\frac{L}{L-1}\left[a_i - \frac{1}{L+M^*-1}\sum_{j\in\mathbf{M}^*} a_j\right]\right\}. \quad (3.4)$$

If there is no $\tilde{M}$ *for which (3.3) is satisfied, then the NE solution is again unique, but assigns zero power level to all* M *users.*

Proof: We first state and prove the following lemma, which will be useful in the proof of the theorem.
Lemma 3.2 *If condition (3.3) is satisfied for* $\tilde{M} = \hat{M}$*, it is also satisfied for all* $\tilde{M}$ *such that* $1 \le \tilde{M} < \hat{M}$*.*
We now show that the solution is strictly positive if, and only if, condition (3.3) is satisfied for $\tilde{M} = M$. First, it is straightforward to show that without the positivity condition (3.2) admits a unique solution. Simple manipulations lead to expression (3.4), with $M^* = M$, for this unique solution. If the NE exists and is strictly positive, then (3.2) has to have a unique positive solution, which we already know is given by (3.4). Hence, (3.4) has to be positive, which is precisely condition (3.3) in view of also the indexing of the users. On the other hand, if (3.3) holds for $\tilde{M} = M$, then we obtain from (3.4) that the equilibrium power level of each user is strictly positive. The existence and uniqueness of the NE follows from (3.2). We thus conclude that-condition (3.3) with $\tilde{M} = M$ is both necessary and sufficient for the existence of a unique positive NE. To complete the proof for the case $M^* = M$, possible boundary solutions need to be investigated to conclude the uniqueness of the inner NE. One has to show that there cannot be another NE, with a subset $\tilde{\mathbf{M}}$ of $\tilde{M}$ users transmitting with positive power, and the remaining $M - \tilde{M}$ users having zero power level. In this case, the reactive power level of the i^{th} mobile, $i \in \tilde{\mathbf{M}}$, is given by (3.4) with $M^* = \tilde{M}$.

For any i^{th} mobile, $i \notin \tilde{\mathbf{M}}$, in order for the zero power level to be part of a NE, condition

$$\bar{y}_{-i} \le L a_i \quad (3.5)$$

should fail according to the reaction function (3.1) of the mobile. Summing up the equilibrium power levels as received by the base station of$\tilde{M}$ users with positive power levels (from (3.4) with $M^* = \tilde{M}$) results in

$$\frac{1}{L}\sum_{j\in\tilde{\mathbf{M}}} y_j = \frac{1}{L+\tilde{M}-1}\sum_{j\in\tilde{\mathbf{M}}} a_j \quad (3.6)$$

Substituting in (3.5), the expression (3.6) for $\bar{y}_{-i}$ yields

$$a_i \ge \frac{1}{L+\tilde{M}-1}\sum_{j\in\tilde{\mathbf{M}}} a_j \; . \quad (3.7)$$

On the other hand, from Lemma 3.2, and (3.3), we have for any i^{th} user in the indexed set $\{1, \ldots, \tilde{M}+1\}$, $a_i \ge \frac{1}{L+\tilde{M}-1}\sum_{j=1}^{\tilde{M}} a_j$. Also, from the indexing of the users it follows that $\sum_{j=1}^{\tilde{M}} a_j \ge \sum_{j\in\tilde{\mathbf{M}}} a_j$. Using this in the previous inequality, we see that (3.7) is satisfied for any i^{th} user, $i \in \{1, \ldots, \tilde{M}+1\}$, regardless of the choice of the subset $\tilde{\mathbf{M}}$. We note that there exists at least one user belonging to the set $\{1, \ldots, \tilde{M}+1\}$, but not the subset $\tilde{\mathbf{M}}$. Thus, the power of that mobile must be positive, and hence the boundary solution cannot be a NE. As this argument is valid for any subset $\tilde{\mathbf{M}}$, all boundary solutions fail similarly for being an equilibrium, including the trivial solution, the origin. Hence the inner NE is unique. This completes the proof for the case $M^* = M$.

If $M^* < M$ in condition (3.3), then the equilibrium will clearly be a boundary point. If condition (3.3) fails for users $M^*+1, \ldots, M$, then they use zero power in the equilibrium. Hence, for any i^{th} user among $M^*+1, \ldots, M$, condition (3.5) should fail. It was shown above that equation (3.6) holds with $\tilde{M} = M^*$. As condition (3.3) does not hold for the i^{th} user, equation (3.7), and hence (3.5) fails. Thus, from (3.1) power level of the i^{th} user is zero, $p_i = 0$, at the equilibrium. As this holds for any $i \in \{\hat{M}+1, \ldots, M\}$, the equilibrium power levels for these users are zero. It can further be shown following similar lines to the ones in the case of $M^* = M$ that the given boundary solution is unique.

Finally, in the case where no M^* exists satisfying condition (3.3), all users fail to satisfy (3.3), and the only solution is the trivial one, $p_i^* = 0 \; \forall i$. ■

4 Update Schemes for Mobiles, and Stability

4.1 Parallel Update Algorithm (PUA)

In the PUA, users optimize their power levels at each iteration (in discrete time intervals) using the reaction function (3.1). If the time intervals are chosen to be longer than twice the maximum delay in the transmission of power level information, it is possible to model the system as an ideal, delay-free one. In a system with delays, there are subsets of users, updating their power levels given the delayed information. The algorithm is

$$p_i^{(n+1)} = \Phi_i(\bar{y}_{-i}^{(n)}, a_i) = \max(0, \frac{1}{h_i}[a_i - \frac{1}{L}\sum_{j\neq i} y_j^{(n)}]) \; , \quad (4.1)$$

whose global stability is established in the next theorem. This means that PUA converges globally to the unique NE of Theorem 3.1 given as

$$p_i^* = \max(0, \frac{1}{h_i}[a_i - \frac{1}{L}\sum_{j\neq i} h_j p_j^*]) \, . \quad (4.2)$$

Theorem 4.1 *PUA is globally stable, and converges to the unique equilibrium solution from any feasible starting point if the following condition is satisfied,* $M-1<L$.

Proof: Let $\triangle y_i^{(n)} := y_i^{(n)} - y_i^*$. Consider first the case when $y_i^* > 0$ for an arbitrary i^{th} user. Then, given $y_j^{(n)}$, $j \neq i$, we have the following from (4.1) and (4.2):

$$|\triangle y_i^{(n+1)}| \begin{cases} < \frac{1}{L}|\sum_{j\neq i} \triangle y_j^{(n)}| \,, & \text{if} \quad a_i < (\bar{y}_{-i}^{(n)}/L) \\ = \frac{1}{L}|\sum_{j\neq i} \triangle y_j^{(n)}| \,, & \text{else} \end{cases}$$

$$\Rightarrow \qquad |\triangle y_i^{(n+1)}| \leq \frac{1}{L}\sum_{j\neq i} |\triangle y_j^{(n)}| \qquad (4.3)$$

Next, we consider the case when $y_i^* = 0$. Then, from (4.1) and (4.2) it follows that

$$|\triangle y_i^{(n+1)}| \leq \begin{cases} \frac{1}{L}|\sum_{j\neq i} \triangle y_j^{(n)}| \,, & \text{if } a_i > (\bar{y}_{-i}^{(n)}/L) \\ 0 & , \quad \text{else} \end{cases}$$

Thus, the inequality (4.3) holds for both cases. Now let $||\triangle y||_\infty := \max_i |\triangle y_i|$. Then, from (4.3),

$$||\triangle y^{(n+1)}||_\infty \leq \frac{1}{L}\max_i \sum_{j\neq i} |\triangle y_j^{(n)}| \leq \frac{M-1}{L}||\triangle y^{(n)}||_\infty \,.$$

Hence, (4.3) is a contraction mapping under condition, $M-1<L$, which leads to the stability and global convergence of the PUA (4.1). ■

4.2 Random Update Algorithm (RUA)

Random update scheme is a stochastic modification of PUA. The users update their power levels with a predefined probability $0 < \pi_i < 1$. Equivalently, at each iteration a set of randomly picked $\pi_i M$ users update their power levels. In the limiting case, $\pi_i = 1$, RUA is the same as PUA. The RUA algorithm is described by

$$p_i^{(n+1)} = \begin{cases} \Phi_i(\bar{y}_{-i}^{(n)}, a_i), & \text{with probability } \pi_i \\ p_i^{(n)}, & \text{with probability } 1-\pi_i \end{cases} ,$$

where Φ_i was defined in (4.1). We already know from the proof of Theorem 4.1 that if user i updates, then (4.3) holds. Hence, for each $i = 1, \dots, M$,

$$E|\triangle y_i^{(n+1)}| \leq \frac{\pi_i}{L}\sum_{j\neq i} E|\triangle y_j^{(n)}| + (1-\pi_i)E|\triangle y_i^{(n)}| \qquad (4.4)$$

Now defining the ℓ_∞ norm $||\triangle y||_\infty := \max_i E|\triangle y_i|$, and following steps as in the case of PUA, we obtain $||\triangle y^{(n+1)}||_\infty \leq (\frac{M-1}{L}\bar{\pi} + (1-\underline{\pi}))||\triangle y^{(n)}||_\infty$, where $\bar{\pi} < 1$ and $\underline{\pi} > 0$ are tight bounds on π_i for all i, that is $\underline{\pi} < \pi_i < \bar{\pi}$. Therefore,

$$\frac{M-1}{L}\bar{\pi} + (1-\underline{\pi}) < 1 \qquad (4.5)$$

is a sufficient condition for the right-hand side of (4.4) to be a contraction mapping, and for the stability and convergence of RUA in norm. We also note that when all users have the same update probability, $\pi_i = \pi \; \forall i$, this condition simplifies to $(M-1)/L < 1$, same sufficient condition as the one for PUA. We show next a stronger result, almost sure (a.s.) convergence of RUA, under condition (4.5). By the Markov inequality and using the definition of the l_∞-norm, we have

$$\sum_{n=1}^{\infty} P(|\triangle y_i^{(n)}| > \varepsilon) \leq \sum_{n=1}^{\infty} \frac{E|\triangle y_i^{(n)}|}{\varepsilon} \leq \frac{1}{\varepsilon}\sum_{n=1}^{\infty} ||\triangle y^{(n)}||_\infty \,,$$

where $P(.)$ denotes the underlying probability measure. Since $E|\triangle y_i^{(n)}|$ is a contracting sequence with respect to the l_∞-norm as shown, $||\triangle y^{(n)}||_\infty \leq \alpha||\triangle y^{(n-1)}||_\infty \leq \cdots \leq \alpha^n||\triangle y^{(0)}||_\infty$, where $0 < \alpha < 1$. Using this in the inequality above, it follows that $\sum_{n=1}^{\infty} P(|\triangle y_i^{(n)}| > \varepsilon) \leq K/(\varepsilon(1-\alpha))$, where $K = ||\triangle y^{(0)}||_\infty$ is a constant. Hence, the increasing sequence of partial sums $\sum_{n=1}^{N} P(|\triangle y_i^{(n)}| > \varepsilon)$ is bounded above by $\frac{K}{\varepsilon(1-\alpha)}$. Thus, it converges for every $\varepsilon > 0$. From the Borel-Cantelli Lemma, it then follows that $P(\limsup_{n\to\infty}\{\omega : |\triangle y_i^{(n)}| > \varepsilon\}) = 0$. Hence, RUA converges also a.s. under condition (4.5).

5 Pricing Strategies at the Base Station

In a noncooperative network, pricing is an important design tool as it creates an incentive for the users to adjust their strategies, in this case power levels, in line with the goals of the network. We will consider in this section two different pricing schemes:

(i) A *centralized pricing scheme:* Users are divided into classes, with all users belonging to a particular class having the same utility function parameter (u_i). Further, all users within a class have the same SIR requirement. The role of the base station is to set prices for these different classes such that, under the resulting NE, the SIR targets of the users are met. A precise result covering the symmetric-user case where every mobile has the same SIR requirement, and $u_i = 1$, is captured by the following theorem.

Theorem 5.1 *Assume that the users are symmetric in their utilities,* $u_i = 1 \; \forall i$*, they have the same minimum SIR requirement,* γ^**, and are charged proportional to their channel gain,* $\lambda_i = kh_i$*. Then the maximum number of users,* M^**, the system can accomodate is bounded* *by $M^* < (L/\gamma^*) + 1$. Moreover, the pricing parameter k under which $M \le M^*$ users achieve the SIR level γ^* is $k = \lambda_i/h_i = (L/\sigma^2)\ (L - \gamma^*(M-1))/(L(\gamma^*+1))$.*

If $M > M^*$, all users fall below the desired SIR level (γ^*) due to the symmetry. In this case, dropping some of the users from the system so as to bring M below M^* would lead to a viable solution. This pricing scheme can be further generalized by splitting the mobiles in a cell into multiple groups according to their desired SIR levels, where the users within each group are symmetric. Using a multiple pricing scheme, a solution capturing these multiple user groups can be obtained.

(ii) *Decentralized, market-based pricing:* The base station sets a single price for all users, and the users choose their willingness to pay parameter (u_i) to satisfy their QoS requirements. As compared to the centralized scheme, this one is more flexible, and allows users to compete for the system resources by adjusting their individual u_i's. After the base station sets an appropriate value for price (k), each user dynamically updates its power level by minimizing its cost under PUA or RUA. As a result, a distributed and market-based power control scheme can be obtained.

The i^{th} mobile can adjust u_i dynamically in accordance with γ_i^*, given the interference at the base station. From (3.1) and (2.2), it follows that $u_i > \frac{\lambda_i}{Lh_i}(\gamma_i^* + 1)(\bar{y}_{-i} + \sigma^2)$. The base station can limit aggressive requests for SIR even in the case when a user pays for its excessive usage of power, by setting an upper-limit, y_{max}, for the received power of the i^{th} user at the base station: $y_i \le y_{max}$. Hence, unresponsive users can be punished by the base station in order to preserve network resources. From (3.1), one can obtain an upper-bound on the value of u_i: $u_i \le \frac{k}{L}[\sigma^2 + (L + M_{max} - 1)y_{max}]\ , \forall i$. When this bound is combined with a simple admission control scheme, limiting the number of mobiles to M_{max}, the base station can provide guarantees for a minimum SIR level, $\gamma_{min} = \frac{L y_{max}}{(M_{max}-1)y_{max}+\sigma^2}$. A tradeoff is observed in the choice of the design parameters γ_{min} versus M_{max}. If the network wants to provide guarantees for a high SIR level, then it has to make a sacrifice by limiting the number of users. In addition, users may implement a distributed admission scheme according to their budget constraints and desired SIR levels. If the necessary price to reach a SIR level exceeds the budget, B_i, of the user, that is $\lambda_i(\gamma_i^* + \bar{y}_{-i} + \sigma^2)/Lu_i h_i \ge B_i$, then the user may simply choose not to transmit at all.

6 Simulation Studies

The power control scheme developed has been simulated using MATLAB, where a discrete time scale was used. The channel gains of users, h_i, were picked randomly with uniform distribution on $[0.2, 1]$.

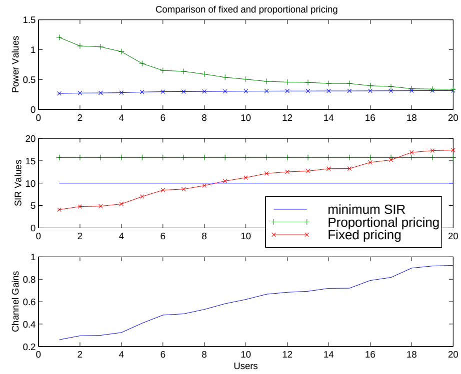


Figure 1: Comparison of power and SIR final values of the mobiles under fixed and proportional pricing.

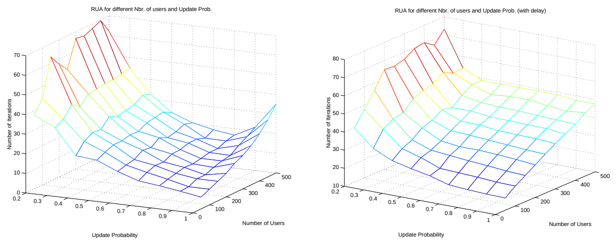
In the first simulation, the two pricing schemes are compared, where users are chosen to be symmetric under both fixed pricing, $\lambda_i = \lambda$, and proportional pricing, $\lambda_i = kh_i$. In Fig. 1, the NE power and the SIR values of each user are depicted under the two pricing schemes. Users with lower channel gains fail to meet the minimum SIR goal, whereas under proportional pricing all users meet it regardless of their channel gain. Proportional pricing is 'fair' in the sense that the users are not affected by their distance to the base station.

The effect of pricing is further investigated in another simulation for a single class of users by varying the pricing parameter, k, under proportional pricing. Equivalently, this simulation can be interpreted as varying the utility parameter, u. From (3.1), the effect of u_i on the system is inversely proportional to k_i. An increase in price leads to decrease in both power and SIR values.

6.1 Convergence Rate and Robustness

We have simulated PUA and RUA for different numbers of symmetric users in both delayed and delay-free cases. The delay factor was introduced into the system in the following way: users are divided into d equal size groups, and each group has an increasing number of units of delay. In Fig. 2(a), the number of iterations to the NE is shown for different probability values of RUA and also for PUA in the delay-free case. As the number of users increase, the optimal update probability decreases. If, however, the number of users is much smaller than the spreading gain, $M \ll L$, then PUA is superior to RUA. When this simulation is repeated in the delayed case, PUA converges faster than RUA for any number of users as shown in Fig. 2(b).

Next, we investigate the robustness of the system in the ideal, delay-free case. First, we analyze it under increasing noise, σ^2. The background noise is increased step by step up to 100% of its initial value. We ob-



(a) (b)

Figure 2: Convergence rate for different π_i's and M's in (a) delay-free (b) delayed cases.

serve in simulations that the power values increase in response to the increasing noise to keep the initial SIR constant. Similarly, when we increase the number of mobiles in the system it has the same effect as increasing the noise due to the nature of CDMA. Thus, all users maintain their SIR levels, confirming the robustness of the power control scheme.

We then simulated the system in a realistic setting where the number of users was modeled as a Markov chain. Arrival of new mobiles was chosen to be Poisson, and call durations were exponentially distributed. A quantity of interest is the average percentage difference between the theoretical equilibrium and the current operating point of the system in terms of power values of users for some period of time. It is observed in the simulations that the system operates within 1% of ideal equilibrium points. Similar results were obtained in another simulation where the channel gains of users were varied randomly up to 15% of their previous values.

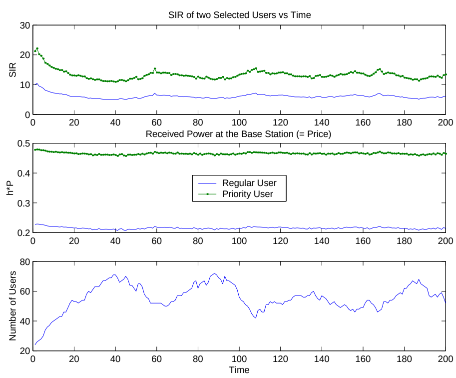


Figure 3: SIR and power levels at the base station of a priority and a regular user versus time.

Finally, a market-based pricing scheme with proportional pricing at the base station is investigated. There are two groups of symmetric users, which have different utility parameters, u. The group with higher u is labeled the "priority" user group, while the other one is called the "regular" user group. In order to observe the effect of variation in the number of users on the SIR levels, we let a sample user from each group make a long enough call. At the same time, the number of users in each group and channel gains of the users were varied as in previous robustness simulations to create realistic disturbances in the system. For simplicity, the values of the utility parameters were kept constant throughout the simulation. In Fig. 3, it is observed that a priority user always obtains a higher SIR than a regular user. The fluctuation in the prices is due to the variation in the number of users and the total demand for SIR.

7 Conclusion

The results obtained for the uplink power control problem in a single cell CDMA wireless network indicate that the game theoretical approach can provide satisfactory decentralized and market-based solutions. There still exist, however, some open questions, which require further investigation. One possible extension of this work is to a multiple cells model, where the effect of neighboring cells are taken into account. Another topic of research is the development of the counterparts of these results in the case of multiple base stations, which brings up the challenging issue of handoff.

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