A Power Control Game Based on Outage Probabilities for Multicell Wireless Data Networks

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***Abstract*— We present a game-theoretic treatment of distributed power control in CDMA wireless systems using outage probabilities. We first prove that the noncooperative power control game considered admits a unique Nash equilibrium (NE) for uniformly strictly convex pricing functions and under some technical assumptions on the SIR threshold levels. We then analyze global convergence of continuous-time as well as discrete-time synchronous and asynchronous iterative power update algorithms to the unique NE of the game. Furthermore, we show that a stochastic version of the discrete-time update scheme, which models the uncertainty due to quantization and estimation errors, converges almost surely to the unique NE point. We finally investigate and demonstrate the convergence and robustness properties of these update schemes through simulation studies.**

***Index Terms* – Power control, communication systems, game theory, code division multiaccess, resource management, stochastic systems.**

I. Introduction

The primary objective of uplink power control in code division multiple access (CDMA) wireless networks is to achieve and maintain a satisfactory level of service, which may be described in terms of signal-to-interference ratio (SIR). Since in CDMA systems signals of other users can be modeled as interfering noise signals, there is a tradeoff between the individual objectives of mobiles and the overall system performance. If mobiles have different preferences for the level of service or varying SIR requirements, then the power control problem can be posed as one of resource allocation. Furthermore, under a distributed power control regime the mobiles cannot have detailed information on each other's preferences and actions due to communication constraints inherent to the system. It is, hence, appropriate to address CDMA uplink power control within a noncooperative game theoretic framework, where Nash equilibrium (NE) provides a relevant solution concept. The power control game can also be extended by making use of pricing. A pricing scheme not only enhances the overall system performance by limiting the interference [1], but also results in battery energy preservation.

Several studies exist in the literature that use game theoretic schemes to address the power control problem [1]–[6]. In [1] a framework for power control based on noncooperative game theory and pricing has been presented. This analysis has then been extended in a later study [3] to multiple cells. The study [4] has shown the existence of unique NE for a certain type of pricing function and under binary input Gaussian output and binary symmetric channel assumptions. Another study [2] has proposed linear and exponential utility functions based on carrier (signal)-to interference ratio, and has shown the existence of a NE under some assumptions on the utility functions. Alpcan et al. [5] has studied a power control game with specific cost structure in a single cell. This analysis has later been extended in [6] to a more general class of cost functions and a multicell framework. In both studies, existence of a unique NE has been proven.

In wireless communication systems, mobiles frequently update their power levels due to varying channel conditions in order to maintain their SIR (service) level. The power control game leads to distributed power control algorithms as a mean to achieve this goal. An important aspect of a distributed power control scheme is the convergence properties of algorithms, which plays a significant role in performance of the system. The study [7] has presented a standard power control algorithm, and has established its synchronous and asynchronous convergence under some conditions on the interference function. In [8], stochastic power control schemes have been investigated, and the converge of stochastic algorithms in terms of mean-squared error

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has been proven. Another study [9] has shown the convergence of a coupled power control scheme based on minimum outage probability and multiuser detection by making use of standard interference functions of [7]. In [5], two update algorithms, namely, parallel update and random update have been shown to be globally stable under specific conditions. Finally, in [6] the global convergence of the dynamics of the power control game to a superset of Nash equilibria has been established for any handoff scheme satisfying a mild condition on average dwell time.

In this paper, we consider a power control game similar to the one in [6], which incorporates a pricing mechanism limiting the overall interference and preserving battery energy of mobiles. We capture the preferences of mobiles using a utility function, which is defined as the logarithm of the probability that the frame success rate of the data user is greater than a predefined individual threshold level. This utility function can also be described in terms of frame outage probabilities [10]. Under the assumption that the fading channel gain (and hence the SIR) is static over a data frame, this frame outage probability can also be related to the standard outage probability notion used for voice communications [9], which is given by the probability that the SIR falls below a predetermined threshold. The notion of an outage probability is useful for rapidly varying fading channels where trying to maintain an SIR threshold may become infeasible or can lead to power warfares. Rapid tracking of the randomly varying channel can also significantly increase the communication overhead between the base station and the mobile transmitter as well. Motivated by these issues, we consider a two-time scale channel gain model, consisting of a slowly varying component which we assume to be constant over the time scale (and hence known) of the application of our algorithm, and a fast fading component which is not known at the transmitter or receiver but we assume that the statistics of this fast fading component are known (or can be accurately estimated. In this paper, we assume that this fast fading component is Rayleigh distributed. For detailed discussions on justifications for using this model, see [9], [11]. In the context of such a generalized fading channel model, we therefore consider a noncooperative power control game which uses an outage probability based (instead of an SIR based) utility function and also incorporates a pricing mechanism.

The noncooperative power control game thus obtained admits a unique Nash equilibrium under uniformly strictly convex pricing functions and some technical assumptions on the SIR threshold levels. Furthermore, we investigate global convergence of continuous-time as well as discrete-time synchronous and asynchronous iterative power update algorithms to the unique NE of the game. A stochastic version of the discrete-time update scheme, which models the uncertainty due to quantization and estimation errors, is shown to converge to the NE almost surely under specific conditions. The convergence and robustness properties of these schemes are demonstrated through simulation studies in MATLAB.

The next section describes the model adopted and the cost function. In section III, we prove the existence and uniqueness of the Nash equilibrium. We present in section IV system dynamics and stability analysis of a continuous-time update scheme. In section V, convergence properties of both deterministic and stochastic discrete-time update algorithms are investigated. Section VI contains results on simulation studies. The paper concludes with a recap of the results and elucidation of directions for future research in section VII.

II. The Model

We consider a multicell CDMA wireless network model similar to the ones described in [3], [9]. The system consists of a set $\mathcal{L} := \{1, \dots, \bar{L}\}$ of cells, with the set of users in cell l being $\mathcal{M}_l := \{1, \dots, M_l\}$, $l \in \mathcal{L}$, and the set of all users is defined as $\mathcal{M} := \bigcup_l \mathcal{M}_l$. The number of users in each cell is limited through an admission control scheme. We associate a single base station (BS) with each cell in the system, and define $h_{il} f_{il} p_i$ as the instantaneous received power level from user i at the l^{th} BS. To simplify the analysis, we let a mobile connect to one BS only at any given time. The quantities h_{il} $(0 < h_{il} < 1)$ and f_{il} $(f_{il} > 0)$ represent the *slow-varying* channel gain (excluding any fading) and fast time-scale Rayleigh fading between the i^{th} mobile and the l^{th} BS, respectively [12]. We assume that the factors affecting h_{il} do not change significantly over the time scale of this analysis, and the terms f_{il} (static over individual data frames but varying from one frame to another) are unit mean independent exponentially distributed random variables (Rayleigh fading).

Let $\mathcal{M}_{l,eff}$ denote the set of users in the neighborhood of cell l who have a nonnegligible effect on each other's SIR levels through in-cell and intra-cell interference. It immediately follows that $\mathcal{M}_l \subset M_{l,eff} \subset \mathcal{M}$. Without loss of any generality, we define the set $\mathcal{M}_{l,eff}$ in this paper as

$$\mathcal{M}_{l,eff} := \mathcal{M}_l \cup \left(\cup_{k \in Neighbor(l)} \mathcal{M}_k\right),$$

where $Neighbor(l)$ is defined as the set of first-tier neighbors of cell l. Furthermore, the contribution of mobiles in other cells to the interference level of cell l is modeled as a fixed background noise, of variance σ_l^2.

The i^{th} mobile transmits with a nonnegative uplink power level of $p_i \leq p_{i,max}$, where $p_{i,max}$ is an upper-bound imposed by physical limitations of the mobile. Thus, in accordance with the interference model considered, the SIR obtained by mobile i at the base station l is given by (static over one data frame)

$$\gamma_{il} := \frac{L h_{il} f_{il} p_i}{\sum_{j \in \mathcal{M}_{l,eff},\, j \neq i} h_{jl} f_{jl} p_j + \sigma_l^2}. \quad (1)$$

Here, $L := W/R > 1$ is the spreading gain of the CDMA system, where W is the chip rate and R is the data rate of the user. The outage probability of user i, denoted O_{il}, is defined as the proportion of time that some SIR threshold, $\bar{\gamma}_{il}$, is not met for sufficient reception at the l^{th} BS receiver [9]. By a careful choice of $\bar{\gamma}_{il}$, a quality of service level can be established for each user (see [10] for details on how a minimum frame success rate can be converted to an appropriate SIR threshold for a specific modulation and coding scheme). The outage probability, $O_{il} = Pr(\gamma_i \leq \bar{\gamma}_{il})$, of the i^{th} mobile at the l^{th} BS is defined as

$$O_{il} = Pr\Bigg(h_{il} f_{il} p_i \leq \bar{\gamma}_{il} \Bigg[\sum_{j \in \mathcal{M}_{l,eff}, j \neq i} h_{jl} f_{jl} p_j + \sigma_l^2 \Bigg]\Bigg), \quad (2)$$

where $Pr(\gamma_i \leq \bar{\gamma}_{il})$ denotes the probability of the event corresponding to $\gamma_i \leq \bar{\gamma}_{il}$.

For analysis purposes, the mean power level of mobile i received at the l^{th} BS can be defined without any loss of generality, as $x_{il} := h_{il} p_i$, since the mean value of the Rayleigh fading channel can be incorporated into the value h_{il}. Let the received power level vector of cell l be $\mathbf{x}_l := [(x_{jl})], j \in \mathcal{M}_{l,eff}$. Then, the systemwide vector $\mathbf{x} := [\mathbf{x}_1, \ldots, \mathbf{x}_{\bar{L}}]$ has the cardinality $Mx := \sum_{l \in \mathcal{L}} M_{l,eff}$, where $M_{l,eff}$ is the number of elements of the set $\mathcal{M}_{l,eff}$. In order to simplify the notation, we will drop the index identifying the BS (e.g. $x_i := x_{il}$) in cases where it is obvious from the context that mobile i is connected to the l^{th} BS. As a further simplification, we let the threshold SIR for the i^{th} mobile be defined as $\bar{\gamma}_i := \bar{\gamma}_{il} = \bar{\gamma}_{ik}\ \forall l, k \in \mathcal{L}$. We note that the outage probability in (2) can be expressed in analytical form which we reproduce here without derivation. Its derivation can be found in [13], and in [11] for a simplified version of the expression. The outage probability of the i^{th} mobile connected to the l^{th} BS is thus given by

$$O_{il}(\mathbf{x}, \bar{\gamma}_i) = 1 - \exp\left(\frac{-\sigma^2 \bar{\gamma}_i}{x_{il}}\right) \prod_{j \in \mathcal{M}_{l,eff}, j \neq i} \frac{1}{1 + \frac{\bar{\gamma}_i x_{jl}}{x_{il}}}, \quad (3)$$

Henceforth we drop the index "l" from x_{il} and O_{il}, and adopt the convention that $j \neq i$ stands for $j \in \mathcal{M}_{l,eff}$, $j \neq i$, where l is the BS to which mobile i is connected.

The i^{th} user's cost function is defined as the difference between the utility function of the user and its pricing function, $J_i = P_i - U_i$, similar to the one in [5]. The utility function, $U_i(Pr_i(\gamma_i \geq \bar{\gamma}_i)$, is a logarithmic function of the probability that the SIR of the i^{th} user is larger than the predefined threshold, $\bar{\gamma}_i$, and quantifies approximately the demand or *willingness to pay* of the user for a certain level of service. Notice that, $Pr_i(\gamma_i \geq \bar{\gamma}_i)$ is equal to $1 - O_i$, where O_i is the outage probability in (3). Hence, the utility function for user i is defined by

$$U_i(\mathbf{x}) := u_i \log(Pr_i(\gamma_i(\mathbf{x}) \geq \bar{\gamma}_i) = u_i \log(1 - O_i(\mathbf{x}, \bar{\gamma}_i)), \quad (4)$$

where u_i is a user-specific utility parameter.

The pricing function, $P_i(p_i)$, on the other hand, is imposed by the system to limit the interference created by the mobile, and hence to improve the system performance [3]. At the same time, it can also be interpreted as a cost on the battery usage of the user. As a result, the cost function of the i^{th} user connected to a specific BS is given by

$$J_i(\mathbf{x}) = P_i(x_i) - u_i \log\left(Pr_i(\gamma_i(\mathbf{x}) \geq \bar{\gamma}_i)\right), \quad (5)$$

where we have used x_i, instead of p_i, as the argument of P_i, by a possible redefinition of the latter.

III. Existence and Uniqueness of Nash Equilibrium

It follows from (4) immediately that the utility function $U_i(\mathbf{x})$ is continuously differentiable in its arguments. In order to calculate the derivatives of the utility function with respect to x, we first evaluate $\partial Pr_i(\gamma_i(\mathbf{x}) \geq \bar{\gamma}_i)/\partial x_i$ using (2) and (3):

$$\frac{\partial Pr_i(\gamma_i(\mathbf{x}) \geq \bar{\gamma}_i)}{\partial x_i} = Pr_i(\gamma_i(\mathbf{x}) \geq \bar{\gamma}_i) \cdot \left(\frac{\sigma_l^2 \bar{\gamma}_i}{x_i^2} + \sum_{j \neq i} \frac{1}{x_i + \frac{x_i^2}{\bar{\gamma}_i x_{jl}}}\right). \quad (6)$$

Thus, the first and second order derivatives of mobile i's utility function, $U_i(\mathbf{x})$, with respect to x_i are given by

$$\frac{\partial U_i(\mathbf{x})}{\partial x_i} = \frac{u_i \sigma_l^2 \bar{\gamma}_i}{x_i^2} + \sum_{j \neq i} \frac{u_i}{x_i + \frac{x_i^2}{\bar{\gamma}_i x_{jl}}} > 0,$$

and

$$\frac{\partial^2 U_i(\mathbf{x})}{\partial x_i^2} = \frac{-2 u_i \sigma_l^2 \bar{\gamma}_i}{x_i^3} - u_i \sum_{j \neq i} \frac{1 + \frac{2x_i}{\bar{\gamma}_i x_{jl}}}{\left(x_i + \frac{x_i^2}{\bar{\gamma}_i x_{jl}}\right)^2} < 0,$$

respectively. Furthermore, for $j \neq i$,

$$\frac{\partial^2 U_i(\mathbf{x})}{\partial x_i \, \partial x_{jl}} = \frac{u_i \bar{\gamma}_i}{(x_i + \bar{\gamma}_i x_{jl})^2} > 0.$$

Let us define x_{min} and x_{max} as lower and upper bounds on x_{il} $\forall i, l$, i.e. $x_{min} < x_{il} < x_{max} \forall i, l$. If the mean received power level of a mobile at the BS is less than x_{min}, then its effect is negligible and it is modeled as part of the background noise. The upper-bound x_{max} is further bounded above by p_{max} with a possible equality in the case of no channel attenuation. We also define $\bar{\gamma}_{min}$ (u_{min}) and $\bar{\gamma}_{max}$ (u_{max}) in such a way that $\bar{\gamma}_{min} < \bar{\gamma}_i < \bar{\gamma}_{max}$ $(u_{min} < u_i < u_{max})$ $\forall i$. We now make the following three assumptions on the price function P_i, for all mobiles i.

A1. The pricing function $P_i(x_i)$ is twice continuously differentiable, non-decreasing and uniformly strictly convex in x_i, i.e.

$$dP_i(x_i)/dx_i \geq 0, \; d^2 P_i(x_i)/dx_i^2 \geq v > 0, \; \forall x_i,$$

for some $v > 0$.

A2. Given the set of parameters $\{M_{l,eff}, \bar{\gamma}_{min}, \bar{\gamma}_{max}, x_{min}, x_{max}\}$, v in A1 above satisfies the following inequality:

$$v(\bar{\gamma}_{min} + 1)\frac{x_{min}^2}{u_{max}} + (M_{l,eff} - 1)\bar{\gamma}_{min} \frac{u_{min}}{u_{max}} \frac{x_{min}^3}{x_{max}^3} > 1$$

A3. The pricing function P_i and the parameter of the utility function are further picked in such a way that the i^{th} user's cost function, J_i, has the following properties at $x_i = x_{min}$ $(x_i = x_{max})$: $\partial J_i(\mathbf{x} : x_i = x_{min})/\partial x_i < 0 \; \forall \mathbf{x}$ $(\partial J_i(\mathbf{x} : x_i = x_{max})/\partial x_i > 0 \; \forall \mathbf{x})$, respectively.

The Nash equilibrium (NE) in a cell is defined as a set of power levels, p^* (and corresponding set of costs J^*), with the property that no user in the cell can benefit by modifying its power level while the other players keep theirs fixed. Mathematically speaking, $\mathbf{x}^*$ is in NE when x_i^* of any i^{th} user is the solution to the following optimization problem given the equilibrium power levels of other mobiles (in the set $\mathcal{M}_{l,eff}$), $\mathbf{x}_{-i}^*$:

$$\min_{x_{min} \leq x_i \leq x_{max}} J_i(x_i, \mathbf{x}_{-i}^*). \tag{7}$$

Note that given the channel gains, the NE point $\mathbf{x}^*$ is equivalent to p^*.

Thanks to assumption *A1*, the cost function J_i is strictly convex and belongs to a fairly large subclass of convex functions. Hence, there exists a unique solution to the i^{th} user's minimization problem, which is that of minimization of J_i, given the system parameters and the power levels of all other users. We will next make use of the technical assumption *A2* in the proof of existence of a unique NE. Notice that, $\mathbf{x}_{min}$ is bounded below by definition. Hence, *A2* is easily satisfied for a large number of users M or high SIR thresholds $\bar{\gamma}_{min}$ even if v is small. Assumption *A3*, on the other hand, ensures that any equilibrium solution is an *inner* one, i.e., boundary solutions $x_i^* = x_{min}$ $(x_i^* = x_{max})$ $\forall i$ cannot be equilibrium points.

Theorem III.1. *Under A1-A3, the multicell power control game defined admits a unique inner Nash equilibrium solution.*

Proof. The proof of this theorem is similar to the ones of Theorem 3.1 in [14] and of Theorem II.1 in [6]. It is briefly outlined here for completeness. Let $X := \{\mathbf{x} \in \mathbb{R}^{Mx} : x_{min} \leq x_{il} \leq x_{max} \; \forall i, l\}$ be a set of feasible received power levels at the base stations under the interference model considered. Clearly, X is closed and bounded, and hence compact. Furthermore, it is also convex, and has a nonempty interior. By a standard theorem of game theory (Theorem 4.4 p.176 in [15]) the power control game admits a Nash equilibrium. In addition, by *A3* this solution has to be inner.

Let $A_{i,j} := \frac{\partial^2 J_i}{\partial x_i \partial x_{jl}}$ and $B_i := \frac{\partial^2 J_i}{\partial x_i^2}$, where mobile i is connected to the BS l. Define $M \times M$ matrix $G(\mathbf{x})$ with diagonal entries B_i and nonzero entries $A_{i,j}$, if $j \in \mathcal{M}_{l,eff}$. It follows from *A2* that $B_i > |A_{i,j}|$ $\forall i, j$. Hence, the symmetric matrix $G(\mathbf{x}) + G(\mathbf{x})^T$ is positive definite. Then, using an argument similar to the one in the proof of Theorem 3.1 in [14] one can show that the inner NE solution is unique. Thus, there exists a unique inner NE in the multicell power control game. □

IV. System Dynamics and Stability Analysis

We consider a dynamic model of the power control game similar to the one of [6] where each mobile uses a gradient algorithm to solve its own optimization problem (7). Accordingly, the power update algorithm of the i^{th} mobile is:

$$\dot{p}_i = \frac{dp_i}{dt} = -\frac{\partial J_i}{\partial p_i},$$

for all $i \in \mathcal{M}$. This can also be described in terms of the received power level, x_i, at the l^{th} BS:

$$\dot{x}_i = h_i^2 \left(\frac{\partial U_i(\mathbf{x})}{\partial x_i} - \frac{dP_i(x_i)}{dx_i} \right) := \phi_i(\mathbf{x}), \ \forall i. \quad (8)$$

By taking the second derivative of x_i with respect to time, we obtain

$$\ddot{x}_i = h_i^2 \left(-a_i - \frac{d^2 P_i(x_i)}{dx_i^2} \right) \dot{x}_i + h_i^2 \sum_{j \neq i} b_{i,j} \dot{x}_{jl} := \dot{\phi}_i(\mathbf{x}), \quad (9)$$

where a_i and $b_{i,j}$ are defined as

$$a_i := -\frac{\partial^2 U_i(\mathbf{x})}{\partial x_i^2} = u_i \frac{2\sigma_l^2 + \bar{\gamma}_i}{x_i^3} + u_i \sum_{j \neq i} \frac{1 + \frac{2x_i}{\bar{\gamma}_i x_{jl}}}{\left(x_i + \frac{x_i^2}{\bar{\gamma}_i x_{jl}} \right)^2},$$

and

$$b_{i,j} := \frac{\partial^2 U_i(\mathbf{x})}{\partial x_i \, \partial x_{jl}} = u_i \frac{\bar{\gamma}_i}{(x_i + \bar{\gamma}_i x_{jl})^2}.$$

Notice that both a_i and $b_{i,j}$ are positive.

We establish the stability of the power update scheme (8) under some sufficient conditions. The set of feasible received power levels is invariant by assumption *A3*, which immediately follows from a boundary analysis. When $x_i = x_{min}$ for some $i \in \mathcal{M}$, we have $\dot{x}_i > 0$ under *A3*. Hence, the system trajectory moves toward inside of X. Likewise, in the case of $x_i = x_{max}$ for some $i \in \mathcal{M}$, $\dot{x}_i < 0$, and hence, the trajectory remains inside the set X. Let us introduce a candidate Lyapunov function $V : \mathbb{R}^{Mx} \to \mathbb{R}$ as

$$V(\mathbf{x}) := \sum_{i \in \mathcal{M}} \frac{1}{h_i^2} \phi_i^2(\mathbf{x}) \ ,$$

which is in fact restricted to the domain X. Note that because of the uniqueness of the NE, $\mathbf{x}^*$, $\phi_i(\mathbf{x}) = 0 \ \forall i$ if and only if $\mathbf{x} = \mathbf{x}^*$. Hence, V is positive for all $\mathbf{x}$ except for $\mathbf{x} = \mathbf{x}^*$.

Taking the derivative of V with respect to t on the trajectories generated by (8), we obtain

$$\dot{V}(\mathbf{x}) \leq \sum_{i \in \mathcal{M}} -(2v + 2a_i)\phi_i^2 + \sum_{i \in \mathcal{M}} \max_j b_{i,j} \sum_{j \neq i} 2|\phi_i \phi_j|.$$

It follows from a simple algebraic manipulation that

$$\sum_{i \in \mathcal{M}} \max_j b_{i,j} \sum_{j \neq i} 2|\phi_i \phi_j| \leq 2(M_{eff}-1) \max_{i,j} b_{i,j} \sum_{i \in \mathcal{M}} \phi_i^2,$$

where $M_{eff} := \max_l M_{l,eff}$.

Using this to bound $\dot{V}$ further yields

$$\dot{V}(\mathbf{x}) \leq (-(2v + \min_i 2a_i) + 2(M_{eff}-1) \max_{i,j} b_{i,j}) \sum_{i \in \mathcal{M}} \phi_i^2.$$

Next, we modify assumption *A2* as follows:

***A2'*.** Assume that the following inequality holds:

$$v(\bar{\gamma}_{min} + 1) \tfrac{x_{min}^2}{u_{max}} + (M_{l,eff} - 1)\bar{\gamma}_{min} \frac{u_{min}}{u_{max}} \frac{x_{min}^3}{x_{max}^3} > M_{eff} - 1 \ \ \forall l.$$

Remark IV.1. *A2'* holds when $\bar{\gamma}_{min}$ and/or v are sufficiently large.

Under *A2'*, we have $\dot{V}(\mathbf{x}) < 0$, uniformly in the x_i's on the trajectory of (8). Thus, V is indeed a Lyapunov function, and it readily follows that $\phi_i(\mathbf{x}(t)) = \dot{x}_i(t) \to 0, \ \forall i$. This in turn implies that $x_i(t)$'s converge to the unique Nash equilibrium. Hence, the unique NE point (Theorem III.1) is globally asymptotically stable on the invariant set X with respect to the update scheme (8) under the assumptions *A1, A2', A3* by Lyapunov's stability theorem (see Theorem 3.1 in [16]).

V. Iterative Power Control Algorithms

We investigate in this section stability properties of synchronous and asynchronous iterative power control schemes as they are of practical importance. We first analyze gradient based synchronous and asynchronous update algorithms of the power control game in Section III. Consequently, we study convergence of stochastic iterations to the unique NE solution by taking communication constraints and estimation errors into account.

A. Synchronous and Asynchronous Update Schemes

Consider a discrete-time counterpart of the update scheme in (8) in a system with M mobiles where each mobile uses a gradient algorithm to solve its optimization problem (7):

$$p_i(n+1) = p_i(n) - \lambda_i \frac{\partial J_i}{\partial p_i} \ \ \forall i \in \mathcal{M} \ ,$$

where $n = 1, 2, \ldots$, denotes the update instances and λ_i is the user-specific step size defined by $\lambda_i := \lambda / h_i$. Here, λ denotes the system wide step size constant. For notational convenience this can also be defined as a mapping from the received power levels at the BS to the updated power levels, $\mathbf{x}(n+1) = T(\mathbf{x}(n))$, i.e.

$$x_i(n+1) = T_i(\mathbf{x}(n)) := x_i(n) - \lambda \frac{\partial J_i}{\partial x_i} \ \ \forall i \in \mathcal{M} \ . \quad (10)$$

In the case of synchronous update algorithm, each mobile updates its power level at the same time instance. We study here sufficient conditions for convergence of the system to the unique NE, $\mathbf{x}^*$, under the synchronous update. This analysis follows lines similar to those in the proof of Proposition 1.10 of [17, p. 193]. Let $\mathbf{x} \in X = \{\mathbf{x} \in \mathbb{R}^{Mx} : x_{min} \leq x_{il} \leq x_{max} \; \forall i, l\}$ and define a function $g_i(\tau) : [0,1] \to \mathbb{R}$ for the i^{th} mobile by

$$g_i(\tau) = \tau x_i + (1-\tau)x_i^* + \lambda \phi_i(\tau \mathbf{x} + (1-\tau)\mathbf{x}^*),$$

where ϕ_i is defined in (8). We then have

$$|T_i(\mathbf{x}) - T_i(\mathbf{x}^*)| = |g_i(1) - g_i(0)| = \left| \int_0^1 \frac{dg_i(\tau)}{d\tau} d\tau \right|$$

$$\leq \int_0^1 \left| \frac{dg_i(\tau)}{d\tau} \right| d\tau \leq \max_{\tau \in [0,1]} \left| \frac{dg_i(\tau)}{d\tau} \right|,$$

where $\mathbf{x}^*$, the NE, is the fixed point of the mapping T. We bound $\left| \frac{dg_i(\tau)}{d\tau} \right|$ above by

$$\left| \frac{dg_i(\tau)}{d\tau} \right| \leq \left| x_i - x_i^* - \lambda \sum_{j \in \mathcal{M}_{l,eff}} \frac{\partial \phi_i}{\partial x_j} \cdot (x_j - x_j^*) \right|$$

$$\leq \left| 1 - \lambda \frac{\partial \phi_i}{\partial x_i} \right| |x_i - x_i^*| + \sum_{j \neq i} \lambda \frac{\partial \phi_i}{\partial x_{jl}} \left| x_{jl} - x_{jl}^* \right|.$$

Imposing the condition $\lambda \partial \phi_i / \partial x_i < 1$, we have

$$\left| \frac{dg_i(\tau)}{d\tau} \right| \leq \left(1 - \lambda \left[\frac{\partial \phi_i}{\partial x_i} - \sum_{j \neq i} \frac{\partial \phi_i}{\partial x_{jl}} \right] \right) \|\mathbf{x} - \mathbf{x}^*\|,$$

where $\|\mathbf{x}\| := \max_i |x_i|$ is the maximum norm. Define

$$K_i := \max_{\mathbf{x} \in X} \frac{\partial \phi_i(\mathbf{x})}{\partial x_i} \text{ and } \rho_i := 1 - \lambda \left(\frac{\partial \phi_i}{\partial x_i} - \sum_{j \neq i} \frac{\partial \phi_i}{\partial x_{jl}} \right),$$

which leads to $|T_i(\mathbf{x}) - x_i^*| \leq \rho_i \|\mathbf{x} - \mathbf{x}^*\|$ for each i. Let $\rho := \max_i \rho_i$ and $K := \max_i K_i$. We obtain then $\|T(\mathbf{x}) - \mathbf{x}^*\| \leq \rho \|\mathbf{x} - \mathbf{x}^*\|$, if $\lambda K < 1$. An upper bound on K in terms of system and cost parameters is

$$\bar{K} := \max_i \frac{d^2 P_i(x_{max})}{dx_i^2} + \frac{2(M_{eff} - 1)\bar{\gamma}_{max} x_{max}}{(\bar{\gamma}_{min} + 1)x_{min}^3} + \frac{2\sigma^2 \bar{\gamma}_{max}}{x_{min}^3}. \quad (11)$$

Imposing the condition $\rho < 1$, it readily follows that for arbitrary $\mathbf{x} \in X$, $T^n(\mathbf{x}) \to \mathbf{x}^*$ as $n \to \infty$, since $\|T^n(\mathbf{x}) - \mathbf{x}^*\| \leq \rho^n \|\mathbf{x} - \mathbf{x}^*\|$. Furthermore, the condition $\rho < 1$ is satisfied if

$$\sum_{j \neq i} \frac{\bar{\gamma}_i^2 x_{jl}^2 + 2\bar{\gamma}_i x_i x_{jl}}{x_i^2 (x_i + \bar{\gamma}_i x_{jl})^2} - \frac{\bar{\gamma}_i}{(x_i + \bar{\gamma}_i x_{jl})^2} > 0 \; \forall i.$$

Let $x_{max} = \alpha x_{min}$ for some $\alpha > 0$. Then, a sufficient condition for $\rho < 1$ is

$$\alpha < 1 + \sqrt{1 + \bar{\gamma}_{min}},$$

which follows from a straightforward algebraic derivation. Thus, under $\lambda \bar{K} < 1$ and $\alpha < 1 + \sqrt{1 + \bar{\gamma}_{min}}$, the synchronous power update scheme given in (10) converges to the NE solution, $\mathbf{x}^*$. This result is summarized in the following theorem:

Theorem V.1. *Let $x_{max} = \alpha x_{min}$ for some $\alpha > 0$ and $X := \{\mathbf{x} \in \mathbb{R}^{Mx} : x_{min} \leq x_{il} \leq x_{max} \; \forall i, l\}$. The synchronous power update algorithm*

$$p_i(n+1) = p_i(n) - \lambda_i \frac{\partial J_i}{\partial p_i} \;\; \forall i \in \mathcal{M}$$

converges to the unique NE point of the power control game, $p^* := [x_1^*/h_1, \ldots, x_M^*/h_M]$, on the set X if

$$\lambda \left[\max_i \frac{d^2 P_i(x_{max})}{dx_i^2} + \frac{2(M_{eff} - 1)\bar{\gamma}_{max} x_{max}}{(\bar{\gamma}_{min} + 1)x_{min}^3} + \frac{2\sigma^2 \bar{\gamma}_{max}}{x_{min}^3} \right] < 1,$$

and

$$\alpha < 1 + \sqrt{1 + \bar{\gamma}_{min}}.$$

Remark V.2. Given x_{min}, x_{max}, α, and system parameters M_{eff} and σ^2, the conditions of Theorem V.1 can be satisfied by choosing λ and $\max_i d^2 P_i(x_{max})/dx_i^2$ sufficiently small while keeping $\bar{\gamma}_{min}$ sufficiently large. We refer to Section VI for specific numerical examples that illustrate this.

A natural generalization of the synchronous update is the asynchronous update scheme where only a random subset of mobiles update their power levels at a given time instance. This is in fact more realistic since it is difficult for the mobiles to synchronize their exact power update instances in a practical implementation. In this particular case, however, the convergence analysis above also applies to the asynchronous update algorithm. Define a sequence of nonempty, convex, and compact sets

$$X(k) := [x_1^* - \delta(k), x_1^* - \delta(k)] \times [x_2^* - \delta(k), x_2^* - \delta(k)] \ldots \times [x_M^* - \delta(k), x_M^* - \delta(k)],$$

where $\delta(k) := \|\mathbf{x}(k) - \mathbf{x}^*\|$. Since by Theorem V.1, $\delta(k+1) < \delta(k)$, we have

$$\ldots \subset X(k+1) \subset X(k) \subset \ldots X.$$

We next give the definitions of two well known conditions which together are sufficient for asynchronous convergence of a nonlinear iterative mapping $\mathbf{x}(n+1) = T(\mathbf{x})$ [17, p. 431].

Definition V.3 (Synchronous Convergence Condition)**.** For a sequence of nonempty sets $\{X(k)\}$ with $\ldots \subset X(k+1) \subset X(k) \subset \ldots X$, we have $T(\mathbf{x}) \in X(k+1), \ \ \forall k$, and $\mathbf{x} \in X(k)$. Furthermore, if $\{y^k\}$ is a sequence such that $y^k \in X(k)$ for every k, then every limit point of $\{y^k\}$ is a fixed point of T.

Definition V.4 (Box Condition)**.** Given a closed and bounded set Y in $\mathbb{R}$, for every k, there exist sets $X_i(k) \subset Y$ such that

$$X(k) := X_1(k) \times X_2(k) \times \cdots \times X_M(k).$$

In our case Y is defined as the interval $[x_{min}, x_{max}]$, and $X_i := [x_i^* - \delta(k), x_i^* + \delta(k)]$. Hence, the box condition is satisfied by the definition of $X(k)$. Since $\delta(k)$ is monotonically decreasing in k by Theorem V.1 the synchronous convergence condition also holds. Therefore, the next convergence result for the asynchronous counterpart of the power update algorithm in (10) immediately follows from asynchronous convergence theorem [17, p. 431].

Theorem V.5. *Let $x_{max} = \alpha x_{min}$ for some $\alpha > 0$ and $X := \{\mathbf{x} \in \mathbb{R}^{Mx} : x_{min} \leq x_{il} \leq x_{max} \ \forall i, l\}$. The asynchronous power update algorithm*

$$p_i(n+1) = \begin{cases} p_i(n) - \lambda_i \frac{\partial J_i}{\partial p_i}, & \textit{if } i \in \mathcal{U}(k) \\ p_i(n), & \textit{if } i \in \mathcal{M} \backslash \mathcal{U}(k), \end{cases}$$

where $\mathcal{U}(k) \subset \mathcal{M}$ denotes the random subset of mobiles updating their power levels at time k, converges to the unique NE point of the power control game, $p^* := [x_1^*/h_1, \ldots, x_M^*/h_M]$, on the set X if

$$\lambda \bar{K} < 1 \quad \textit{and} \quad \alpha < 1 + \sqrt{1 + \bar{\gamma}_{min}}\,,$$

where $\bar{K}$ is defined in (11).

B. A Stochastic Update Scheme

In a real life implementation of the power control scheme, communication constraints, approximations, estimation and quantization errors may not be negligible, and hence have to be taken into account in the convergence analysis. Hence, a mobile does not have access to the exact values of the system parameters such as its own channel gain or the feedback terms provided by the BS. These uncertainties can be captured by a stochastic update algorithm, as introduced below. For each $i \in \mathcal{M}$, let $\xi_i(n) \ n = 1, 2, \ldots$ be a sequence of independent identically distributed (i.i.d.) random variables defined on the common support set $[1-\varepsilon, 1+\varepsilon]$, where $0 < \varepsilon < 1$. We further assume that the sequences $\{\xi_i\}$ are independent across $i \in \mathcal{M}$. Using these random sequences, we model the aggregate uncertainty in the term $\partial J_i / \partial p_i$ of (10) due to quantization, estimation, and multiplicatively approximation errors. Thus, the stochastic counterpart of the synchronous update algorithm is given by

$$p_i(n+1) = p_i(n) - \lambda_i \xi_i(n) \frac{\partial J_i}{\partial p_i} \quad \forall i \in \mathcal{M}, \qquad (12)$$

which can also be described in terms of received power levels at the base station as

$$\begin{aligned} x_i(n+1) &= x_i(n) - \lambda \xi_i(n) \frac{\partial J_i}{\partial x_i} \\ &=: T_i(\mathbf{x}(n);\, \xi_i(n)) \ \ \forall i \in \mathcal{M}. \end{aligned} \qquad (13)$$

We next follow steps similar to those in the previous subsection for the convergence analysis. We have, for an arbitrary $\mathbf{x} \in X$:

$$E\left(|T_i(\mathbf{x};\xi_i) - x_i^*|\right) \leq E\Bigg(\left|1 - \lambda \xi_i \frac{\partial \phi_i}{\partial x_i}\right| |x_i - x_i^*| + \textstyle\sum_{j \in \mathcal{M}_{l,eff},\, j \neq i} \lambda \xi_i \frac{\partial \phi_i}{\partial x_{jl}} \left| x_{jl} - x_{jl}^* \right| \Bigg),$$

where $E(x)$ denotes the expected (mean) value of x. Assume $\lambda(1+\varepsilon)K_i < 1$, where K_i, as defined earlier, provides an upper bound on $\partial \phi_i / \partial x_i$. Then, from the independence of ξ_i and x_i for all i, we obtain (by dropping the dependence on n):

$$E\left(|T_i(\mathbf{x};\xi_i) - x_i^*|\right) \leq (1 - \lambda E(\xi_i) K_i') E\left(|x_i - x_i^*|\right) + \lambda E(\xi_i) \bar{K}_i \textstyle\sum_{j \neq i} E\left(\left| x_{jl} - x_{jl}^* \right| \right),$$

where K_i' is a lower bound on $\partial \phi_i / \partial x_i$, and $\bar{K}_i$ is an upper bound on $\partial \phi_i / \partial x_j$ for all $j \neq i$. Let us redefine the maximum norm as $\|\mathbf{x}\| = \max_i E(|x_i|)$. Then, $E\left(|T_i(\mathbf{x};\xi_i) - x_i^*)|\right) \leq \bar{\rho}_i \|\mathbf{x} - \mathbf{x}^*\| \ \ \forall i$, where $\bar{\rho}_i := 1 - \lambda E(\xi_i)(K_i' - (M_{eff} - 1)\bar{K}_i)$. Defining $\bar{\rho} := \max_i \rho_i$, we obtain

$$\|T(\mathbf{x};\xi) - \mathbf{x}^*\| \leq \bar{\rho} \|\mathbf{x} - \mathbf{x}^*\|,$$

if $\lambda(1+\varepsilon)\bar{K} < 1$, where $\xi := [\xi_1, \xi_2, \ldots, \xi_M]$. Now, imposing the condition $\bar{\rho} < 1$, it readily follows that for arbitrary $x \in X$ and $\xi_i(n) \in [1-\varepsilon, 1+\varepsilon]$ $\forall i$, n, we have $T^n(\mathbf{x};\xi) \to \mathbf{x}^*$ as $n \to \infty$, since $\|T^n(\mathbf{x};\xi) - \mathbf{x}^*\| \leq \bar{\rho}^n \|\mathbf{x} - \mathbf{x}^*\|$. We note that the condition $K_i' > (M_{eff} - 1)\bar{K}_i \;\; \forall i$ is equivalent to the one $\bar{\rho} < 1$. Hence, a derivation similar to the one in the deterministic case yields a sufficient condition for $\bar{\rho} < 1$ to hold, namely

$$\alpha < \frac{1}{2}\sqrt{\bar{\gamma}_{min}} + \frac{1}{4},$$

where α is defined as before with x_{max} and x_{min} being upper and lower bounds on the random variables x_i for all i.

We next show that the stochastic update scheme (13) converges almost surely (a.s.) [18] to the unique NE solution $\mathbf{x}^*$, under the given conditions, by an analysis similar to the one in [5]. From the Markov inequality and using the definition of the maximum norm, we obtain

$$\sum_{n=1}^{\infty} P(|x_i(n)| > \epsilon) \leq \sum_{n=1}^{\infty} \frac{E(|x_i(n)|)}{\epsilon}$$
$$\leq \frac{1}{\epsilon}\sum_{n=1}^{\infty} \|\mathbf{x}(n)\| \leq \frac{1}{\epsilon}\sum_{n=1}^{\infty} \bar{\rho}^n \|\mathbf{x}(0)\| \leq \frac{\|\mathbf{x}(0)\|}{\epsilon(1-\bar{\rho})},$$

where $\epsilon > 0$ and $\|\mathbf{x}(0)\|$ are constants, $Pr(A)$ denotes the probability of the event A, and the last inequality follows from the contraction property of the normed random sequence. Hence, the increasing sequence of partial sums $\sum_{n=1}^{N} Pr(|x_i(n)| > \epsilon)$ is bounded above, and converges for every $\epsilon > 0$. Finally, from the Borel-Cantelli lemma [19], [20], it follows that

$$Pr(\limsup\{\omega : |x_i(\omega)| > \epsilon\}) = 0 \;\forall i,$$

where ω is the probabilistic variable. Thus, the stochastic update scheme (13) converges a.s. to the unique NE point of the power control game under the conditions $\bar{\rho} < 1$ and $\lambda(1+\varepsilon)K < 1$.

Theorem V.6. *Let $\mathbf{x}_i(n)$ $(\xi_i(n))$ be random (random i.i.d.) sequences for all i, where ξ_i is also independent across i and has the support set $[1-\varepsilon, 1+\varepsilon]$, $0 < \varepsilon < 1$. The random vector $\mathbf{x}$ takes values in the set $X := \{\mathbf{x} \in \mathbb{R}^{Mx} : x_{min} \leq x_{il} \leq x_{max} \;\forall i, l\}$. Furthermore, let $\alpha > 0$ be defined as $\alpha := x_{max}/x_{min}$. The stochastic power update algorithm*

$$p_i(n+1) = p_i(n) - \lambda \xi_i(n) \frac{\partial J_i}{\partial p_i} \;\; \forall i \in \mathcal{M},$$

converges almost surely to the unique NE point of the power control game, p^*, if

$$\alpha < \frac{1}{2}\sqrt{\bar{\gamma}_{min}} + \frac{1}{4} \;\; \textit{and} \;\; \lambda(1+\varepsilon)\bar{K} < 1$$

where $\bar{K}$ is defined in (11).

VI. SIMULATIONS

The power control game based on outage probabilities is simulated in MATLAB for a wireless network consisting of 6 arbitrarily placed base stations and 20 mobiles. The channel gain of the i^{th} mobile is determined by the Rayleigh fast-fading and log-normal shadowing path loss model, given by $g_i = (0.1/d_i)^{2.5} \cdot Y_\sigma^{-1} \cdot f_i$, where d_i denotes the distance to the BS, $\log(Y_\sigma)$ is a zero-mean Gaussian random variable with a standard deviation of $\sigma = 0.1$, and f_i is a random variable with Rayleigh distribution, modeling the fast-fading channel. We generate the random variable f_i at each time step and Y_σ every 20 time steps according to their respective distributions. The distance based loss exponent is chosen as 2.5, which corresponds to a low density urban environment [12]. Each mobile connects to a single BS, which happens to be in the closest geographical location. Hence, the cells in the network are irregularly shaped polygons. The system parameters are chosen as $L = 128$ and $\sigma_l^2 = 0.1 \;\forall l$.

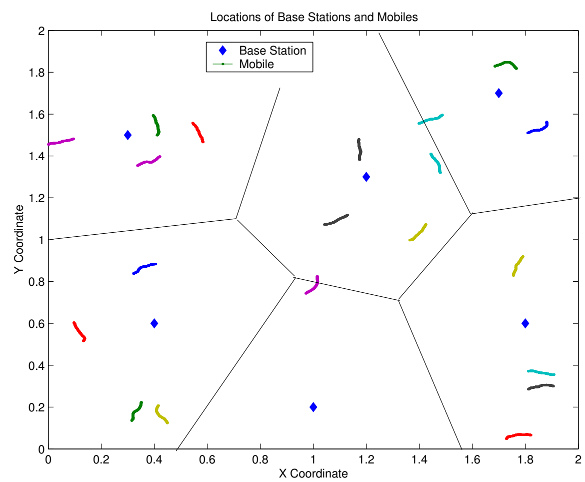

Fig. 1. Locations of base stations and the paths of mobiles.

The mobiles are initially distributed randomly over the network, and their movement is modeled after a two-dimensional random walk with a speed of 0.0001 units per update. In order to relate the values of the simulation to real physical quantities, we assume an update frequency of $1kHz$ and geographical unit size of $100m$. Thus, mobiles move with a speed of $10m/s$ or $36km/h$. We note, however, that these are arbitrarily fixed values, for illustration purposes only. Figure 1 depicts the locations of the BSs and the paths of all mobiles.

The class of user pricing functions which satisfy the earlier convexity assumptions is fairly large. The relationship between the pricing function and the performance of the system at the NE point is in fact a very complex one, and therefore the question of finding the "optimum" pricing function, though interesting, does not seem to be within reach. Consequently, we adopt a specific one without any optimality consideration; namely we choose a quadratic function parametrized by v_i for the i^{th} user as a representative pricing function in our numerical studies. Thus, the cost function for the i^{th} user (mobile) is

$$J_i(\mathbf{x}) = \frac{1}{2} v_i x_i^2 - u_i \log(Pr_i(\gamma_i(\mathbf{x}) \geq \bar{\gamma}_i)),$$

where pricing and utility parameters are $u_i = 10$, $v_i = 1$, and $\bar{\gamma}_i = 10\,(10dB)$, which are chosen to be the same for all users for comparison purposes.

We first simulate a discrete update scheme with "perfect" information where we ignore the communication constraints between the BS and the mobiles. In order to estimate the slow varying $x_i\,(= h_i p_i)$ value of the i^{th} mobile, the BS implements a maximum likelihood estimator (MLE) using the last 20 independent identically exponentially distributed samples of the received power level $[g_i^{(1)} p_i, g_i^{(2)} p_i, \ldots, g_i^{(20)} p_i]$. Here, we consider a sufficiently high sampling frequency so that we can assume p_i to be constant within an interval of 20 samples. A straightforward derivation of this unbiased MLE yields

$$h_i p_i = \sqrt{\frac{\pi}{4 \cdot 20} \sum_{k=1}^{20} \left(g_i^{(k)} p_i \right)^2}.$$

The output of this estimator is then filtered with a simple infinite impulse response (IIR) low pass filter (LPF) to cancel out the effect of high frequency estimation errors and other disturbances. Figure 2 depicts the instantenous and filtered estimation channel gains from mobile 1 to its BS. Thus, given the feedback information from the BS, the mobiles update their power levels according to

$$\begin{aligned} p_i(n+1) &= p_i(n) + \lambda u_i \frac{\sigma_l^2 \bar{\gamma}_i}{h_{il}^2 p_i^2(n)} \\ &+ \frac{\lambda u_i}{h_{il} p_i(n)} \sum_{j \neq i} \frac{1}{1 + \frac{h_{il} p_i(n)}{h_{jl} p_j(n) \bar{\gamma}_i}} - \lambda v_i h_i p_i(n), \end{aligned} \quad (14)$$

where $\lambda = 0.1$ and n denotes the time, and mobile i is connected to the l^{th} BS.

The power levels and SIR values of a randomly selected subset of mobiles for the duration of the simulation are shown in Figures 3 and 4, respectively. The

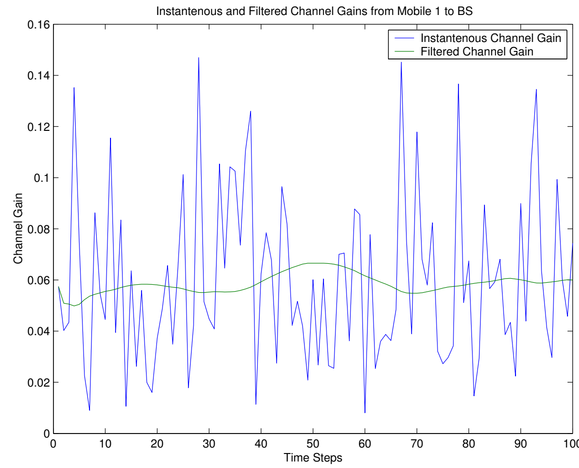

Fig. 2. Instantenous and filtered channel gain from mobile one to its respective BS.

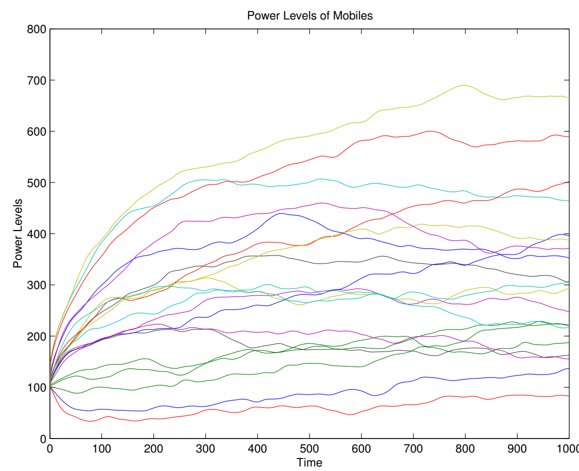

Fig. 3. Power levels of selected mobiles with respect to time.

average SIR values in Figure 4 are obtained by using the filtered channel gains of mobiles instead of instantenous ones. They are provided in order to visualize the trends in SIR values. The minimum and maximum received power levels of the mobiles at their respective BSs are $x_{min} = 2.5$ and $x_{max} = 90$. Hence, we obtain $\alpha = x_{max}/x_{min} = 36$. Figure 5 depicts the evolution of the received power levels of selected mobiles at their respective BSs. While these parameters satisfy assumption **A2**, they violate assumption **A2'** as well as conditions of Theorem V.1. Since the derived analytical conditions in previous sections were only sufficient, and not necessary, it is not surprising that the power levels still converge to the equilibrium points which slowly shift due to the movements of the mobiles.

In the next simulation, we change the SIR threshold value of mobiles to $\bar{\gamma}_i = 1000\,(30dB)$ and let $\lambda = 0.01$.

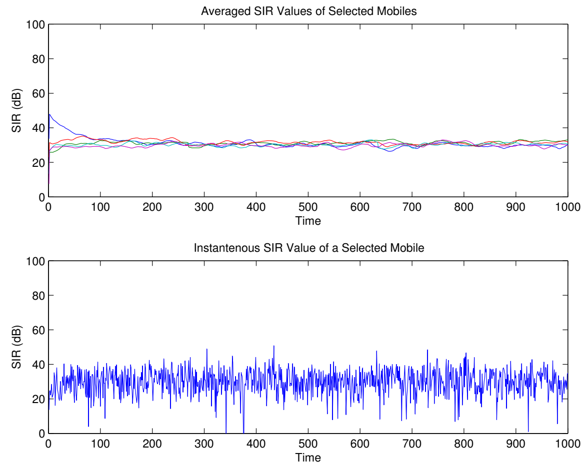

Fig. 4. SIR and averaged SIR values of selected mobiles (in dB) with respect to time.

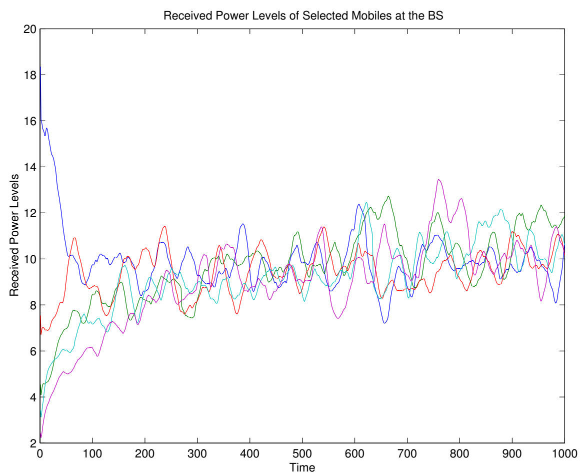

Fig. 5. The received power levels of selected mobiles at their respective BSs.

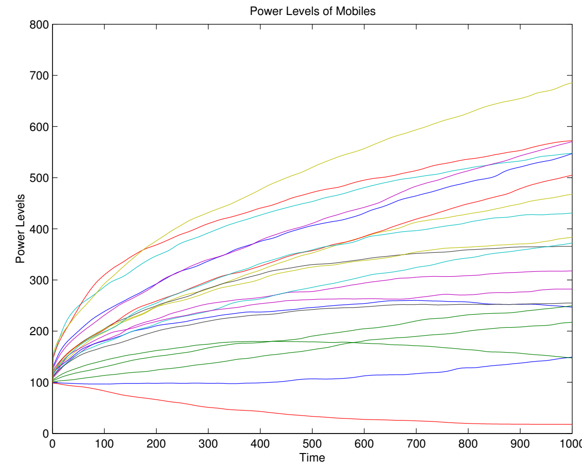

Fig. 6. Power levels of selected mobiles with respect to time.

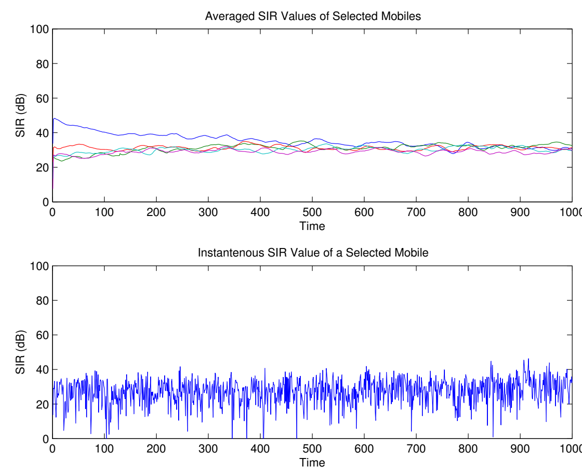

Fig. 7. SIR and averaged SIR values of selected mobiles (in dB) with respect to time.

Furthermore, we have $x_{min} = 3$ and $x_{max} = 48$, and hence, $\alpha = 16$. It is easy to see that these parameters satisfy assumptions **A2** and **A2'**, and the conditions of Theorem V.1. The results in Figures 6 and 7 show convergence as expected. However, we observe that the convergence speed in this case is slower due to the smaller step size. We conclude that although the sufficient conditions derived analytically provide a guideline for the convergence of the algorithm, they are by no means necessary and may be too stringent in some cases.

We next consider a more realistic information feedback scheme, where we take into account the distortion in feedback information due to quantization and other effects. Multiplying the parameter $\lambda = 0.1$ in the update algorithm (14) with ξ, which is a random variable uniformly distributed on $[0.7, 1.3]$, we rerun the previous simulation with this imperfect feedback algorithm. Figures 8 and 9 depict respectively the power levels and SIR values of selected mobiles. In accordance with Theorems V.1 and V.6, the convergence characteristics of the system are not significantly affected. We finally study the effect of the pricing parameter v on the overall performance of the system. We calculate the sum of the utility values of static arbitrarily located mobiles for $u = 5$. Figure 10 displays the sum of the utility values of mobiles averaged over the fast fading process at the NE solution. After repeating this analysis several times for various distributions of mobiles, we conclude that there is a complex and nonlinear relationship between the NE point and the pricing parameter v, which can be interpreted as the cost on the battery usage of the user.

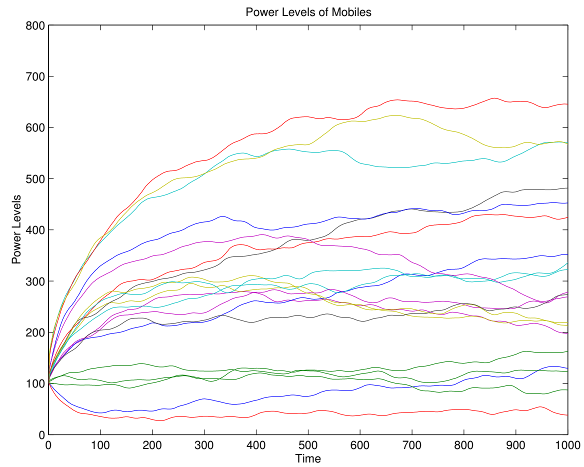

Fig. 8. Power levels of selected mobiles with respect to time under imperfect feedback information.

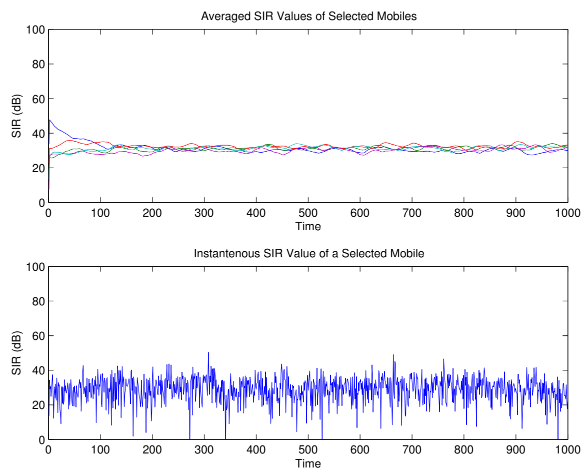

Fig. 9. SIR and averaged SIR values of selected mobiles (in dB) with respect to time under imperfect feedback information.

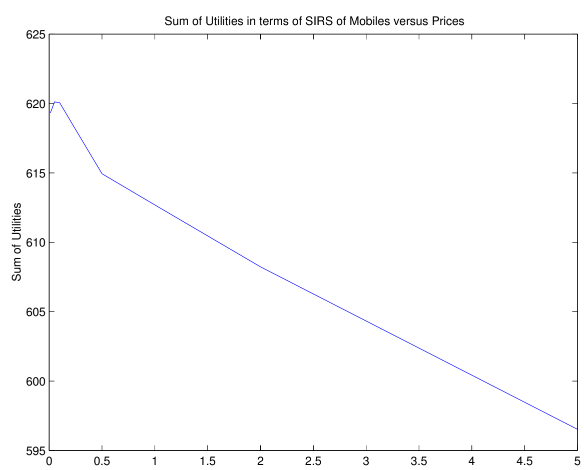

Fig. 10. Sum of the utility values of mobiles for different v values.

VII. Conclusions

In this paper, we have considered a power control game similar to the one in [6], with a utility function defined as the logarithm of the probability that the SIR level of the mobile is greater than a predefined individual threshold level. Hence, we have established a relationship between the preferences of the mobiles and outage probabilities. We have proven that the noncooperative power control game admits a unique Nash equilibrium for uniformly strictly convex pricing functions and under some technical assumptions on the SIR threshold levels. Furthermore, we have established the global convergence of continuous-time as well as discrete-time synchronous and asynchronous iterative power update algorithms to the unique NE of the game under some conditions. Likewise, a stochastic version of the discrete-time synchronous update scheme, which accounts for the uncertainty due to quantization and estimation errors, has been shown to converge to the unique NE point almost surely. Finally, through extensive simulation studies we have demonstrated the convergence and robustness properties of power update schemes developed.

A possible extension of this study would involve the simulation of asynchronous update schemes as well as analysis and simulation of various handoffs algorithms. Another research direction would be the exploration of the relationship between the pricing function and system performance, and its investigation as an optimization problem.

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