A Hybrid Systems Model for Power Control in Multicell Wireless Data Networks ⋆

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Abstract

We present a power control scheme based on noncooperative game theory, using a fairly broad class of convex cost functions. The multicell CDMA wireless data network is modeled as a switched hybrid system where handoffs of mobiles between different cells correspond to discrete switching events between different subsystems. Under a set of sufficient conditions, we prove the existence of a unique Nash equilibrium for each subsystem, and prove global exponential stability of an update algorithm. We also establish the global convergence of the dynamics of the multicell power control game to a convex superset of Nash equilibria for any switching (handoff) scheme satisfying a mild condition on average dwell-time. Robustness of these results to feedback delays as well as to quantization is investigated. In addition, we consider a quantization scheme to reduce the communication overhead between mobiles and the base stations. Finally, we illustrate the power control scheme developed through simulations.

Key words: Wireless networks, Power control, Hybrid systems, Noncooperative game theory, Nash equilibrium

1 Introduction

The primary objective of power control in a wireless network is to regulate the transmission power level of each mobile in order to obtain and maintain a satisfactory quality of service for as many users as possible. In a code division multiple access (CDMA) system, where signals of other users can be modeled as interfering noise, the goal of power control is more precisely stated as to achieve a certain signal to interference (SIR) ratio regardless of channel conditions while minimizing

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the interference, and hence improving the overall performance. Although there exists a large body of work for voice traffic where SIR requirements for satisfactory service are fixed and well established, power control for wireless data networks has only recently been a topic of interest [1–7]. Since the SIR requirements for a desired level of service vary from one individual user to another in wireless networks, the power control problem becomes also one of resource allocation. Recent studies [1,4,5] make use of concepts and tools from the field of economics, such as pricing and utility functions, to come up with power control schemes that address this question. In [4], a pricing scheme for the downlink of a wireless network is investigated where users are charged based on their channel quality. The study [5], on the other hand, shows that net utility maximization problem for elastic traffic can be decomposed into simpler problems of obtaining the optimal signal quality and selection of the optimal transmission rate.

In CDMA systems where each mobile interacts with others by affecting the SIR ratio through interference, game theory provides a natural framework for analyzing and developing power control mechanisms. For a mobile in such a network, obtaining individual information on the power level of each of the other users is practically impossible due to the excessive communication and processing overhead required. Therefore in a distributed power control setting each user attempts to minimize its own cost (or maximize its utility) in response to the aggregate information on the actions of the other users. This makes use of noncooperative game theory [8] for uplink power control most appropriate, with the relevant solution concept being the noncooperative Nash equilibrium [9].

Several studies exist in the literature, which use game theoretic schemes to address the power control problem in a single cell [3,1,10]. In [9], we have made use of the conceptual framework of noncooperative game theory to obtain a distributed and market-based control mechanism. We have proven the existence of a unique Nash equilibrium, and established the stability of two different update algorithms under some specific conditions. The study [10] defines a similar single cell power control game with a new pricing function, which admits a unique Nash equilibrium. In addition, it studies the asymptotic behavior of the system. Another recent study, [6], investigates pricing and power control in a multicell wireless network. Here, existence of a unique Nash equilibrium for a class of quasiconcave utility functions is established without pricing. The effect of linear pricing schemes on the solutions are also analyzed and it is shown that pricing improves Pareto efficiency of the operating (equilibrium) points.

In this paper, we extend the single cell power control scheme of [9] to multiple cells and to a broader class of cost functions. Specifically, we model the multicell wireless data network as a switched hybrid system where handoffs of mobiles between the individual cells (base stations) correspond to discrete switching events between different subsystems. Under a set of sufficient conditions, we show in Section 2 the existence and global stability of a unique Nash equilibrium for each subsystem. In addition, we consider here a more realistic interference model than the one in [11], which is an earlier conference version of this paper, by taking intra-cell effects into account. In Section 3, we establish the global exponential convergence of the dynamics of the multicell power control game to a minimum convex set of Nash equilibria for any switching (handoff) scheme satisfying a mild condition on average dwell-time. Furthermore, we investigate robustness of these results to various communication constraints such as feedback delays and quantization. In addition, we analyze a quantization scheme to reduce the communication overhead between mobiles and the base stations. Finally, we illustrate the proposed power control scheme through MATLAB simulations in Section 4, which is followed by the concluding remarks of Section 5.

2 The Model and Nash Equilibrium

We consider a multicell CDMA wireless network model similar to the ones described in [6,9]. The system consists of a set $\mathcal{L} := \{1, \ldots, \bar{L}\}$ of cells, with M_l users in cell l, $l \in \mathcal{L}$. The number of users in each cell is limited through an admission control scheme. We let $\mathcal{M}_l := \{1, \ldots, M_l\},\ l \in \mathcal{L}$ and $\mathcal{M} := \cup_l \mathcal{M}_l$. We associate a single base station (BS) with each cell in the system, and define $0 < h_{ij} < 1$ as the channel gain between the i-th mobile and the j-th BS. In each cell l, we consider a background noise of a cell specific variance σ_l^2. Furthermore, we assume that each mobile is connected to a single BS at any given time. Figure 1 shows a simple depiction of the wireless network model considered.

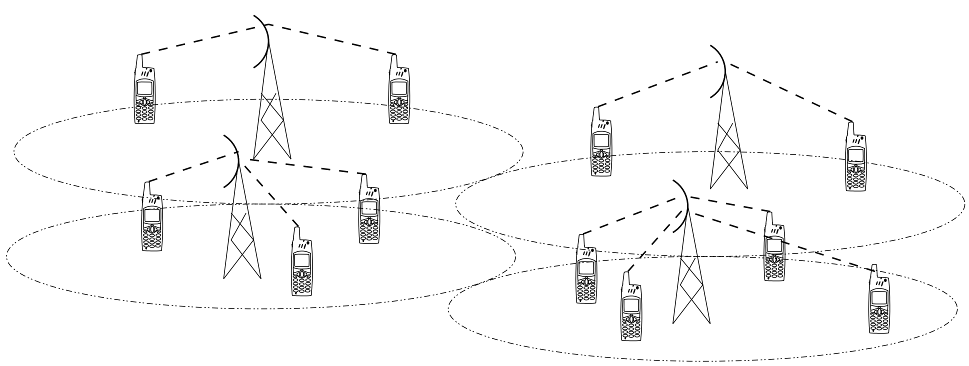

Fig. 1. A simple multicell wireless network.

The i-th mobile transmits with an uplink power level of $p_i \leq p_{i,max}$, where $p_{i,max}$ is an upper-bound imposed by physical limitations of the mobile. Thus, the SIR obtained by mobile i at the base station j is given by

$$\gamma_{ij} := \frac{L h_{ij} p_i}{\sum_{k \in \mathcal{M},\, k \neq i} h_{kj} p_k + \sigma_j^2}. \tag{1}$$

Here, $L := W/R > 1$ is the spreading gain of the CDMA system, where W is the chip rate and R is the data rate of the user. Thus, by taking the intra-cell effects into account we consider a more complex and realistic interference model than the one in [11].

Define the power level of mobile i received at the j-th BS as $x_{ij} := h_{ij}p_i$, and let $\mathbf{x} := [x_{11}, \ldots, x_{M_1 1}, \ldots, x_{1L}, \ldots, x_{M_L L}]$. Also let $\mathbf{x}_{-i} := \sum_{k \in \mathcal{M},\, k \neq i} h_{kj}p_k$ be the sum of the received power levels of all mobiles, except the i-th one, at the j-th BS. In order to simplify the notation we will drop the index of the BS (e.g. $x_i := x_{ij}$) in cases when it is obvious from the context that mobile i is connected to the j-th BS.

The i-th user's cost function is defined as the difference between the utility function of the user and its pricing function, $J_i = P_i - U_i$, similar to the one in [9]. The utility function, $U_i(\gamma_i)$, is a function of the SIR, γ_i, of the i-th user, and quantifies approximately the demand or *willingness to pay* of the user for bandwidth. The pricing function, $P_i(p_i)$, on the other hand, is imposed by the system to limit the interference created by the mobile, and hence to improve the system performance [6]. At the same time, it can also be interpreted as a cost on the battery usage of the user. As a result, the cost function of the i-th user connected to a specific BS is given by

$$J_i(x_i, \mathbf{x}_{-i}, h_i) = P_i(x_i) - U_i(\gamma_i(\mathbf{x}))\,, \tag{2}$$

where we have used x_i, instead of p_i, as the argument of P_i, by a possible redefinition of the latter. We now make the following assumptions on the cost functions:

A1. $P_i(x_i)$ is twice continuously differentiable, non-decreasing and strictly convex in x_i, i.e.

$$\partial P_i(x_i)/\partial x_i \geq 0,\, \partial^2 P_i(x_i)/\partial x_i^2 > 0,\, \forall x_i.$$

A2. $U_i(\gamma_i(\mathbf{x}))$ is jointly continuous in all its arguments and twice continuously differentiable, nondecreasing and strictly concave in x_i, i.e. $\partial U_i(\mathbf{x})/\partial x_i \geq 0$, $\partial^2 U_i(\mathbf{x})/\partial x_i^2 < 0$, $\forall x_i$.

A3. $U_i(\gamma_i)$ satisfies the inequality

$$\left|\frac{d^2 U_i}{d\gamma_i^2}\right| \gamma_i < \frac{dU_i}{d\gamma_i} < (L + \gamma_i)\left|\frac{d^2 U_i}{d\gamma_i^2}\right|,$$

where $|.|$ denotes the absolute value function.

A4. The i^{th} user's cost function has the following properties: At $x_i = 0$, $J_i(\mathbf{x} : x_i = 0) > J_i(\mathbf{x})\ ,\forall \mathbf{x}\ x_i \neq 0$, and at $x_{i,max} = x_{i,max}$, $J_i(\mathbf{x} : x_i = x_{i,max}) > J_i(\mathbf{x})\ ,\forall \mathbf{x}\ x_i < x_{i,max}$, respectively.

Thanks to assumptions *A1* and *A2* the cost function J_i is strictly convex, and belongs to a fairly large subclass of convex functions. Hence, there exists a unique solution to the i-th user's minimization problem, which is that of minimization of J_i, given the system parameters and the power levels of all other users. The Nash equilibrium (NE) is defined as a set of power levels, $\mathbf{p}^*$ (and corresponding set of costs J^*), with the property that no user can benefit by modifying its power level while the other players keep theirs fixed. Furthermore, assumption *A4* ensures that any equilibrium solution is an *inner* one, i.e., boundary solutions $x_i^* = 0 \ (x_i^* = x_{i,max}) \ \ \forall i$ cannot be equilibrium points. Mathematically speaking, $\mathbf{x}^*$ is in NE, when x_i^* of any i-th user is the solution to the following optimization problem given the equilibrium power levels of other mobiles, $\mathbf{x}^*_{-i}$:

$$\min_{0 \le x_i \le x_{i,max}} J_i(x_i, \mathbf{x}^*_{-i}, h_i). \tag{3}$$

Theorem 1 *Under A1-A4, the multicell power control game admits a unique inner Nash equilibrium solution.*

PROOF. The proof of this theorem is similar to the one of Theorem 3.1 in [12]. It is briefly outlined here for completeness. Let $X := \{\mathbf{x} \in \mathbb{R}^M : 0 \le \mathbf{x} \le \mathbf{x}_{max}\}$ be a set of feasible received power levels at the base stations in the system. Clearly, X is closed and bounded, and hence compact. Furthermore, it is also convex, and has a nonempty interior. By a standard theorem of game theory (Theorem 4.4 p.176 in [8]) the power control game admits a Nash equilibrium. In addition, by *A4* this solution has to be inner. It follows from *A3* that

$$\frac{\partial^2 J_i}{\partial x_i^2} > \frac{\partial^2 J_i}{\partial x_i \partial x_j} > 0.$$

Finally, using an argument similar to the one in the proof of Theorem 3.1 in [12] one can show that the inner NE solution is unique. Thus, there exists a unique inner NE in the multicell power control game. □

3 Hybrid Modeling and Stability

We consider a dynamic model of the power control game where each mobile uses a gradient algorithm to solve its own optimization problem (3). The update scheme of the i-th mobile is given by

$$\dot{p}_i = \frac{dp_i}{dt} = -\lambda_i \frac{\partial J_i}{\partial p_i},$$

where $\lambda_i > 0$ is a user-dependent stepsize. This can also be described in terms of the received power level, x_i, at the BS l to which mobile i is connected:

$$\dot{x}_i = \frac{dU_i}{d\gamma_i} \frac{L \lambda_i h_i^2}{\sum_{j \ne i} (h_{jl}/h_j) x_j + \sigma_l^2} - \lambda_i h_i \frac{dP_i}{dp_i} := \phi_i(\mathbf{x}), \tag{4}$$

where h_j denotes the channel gain of mobile j to its own BS. Thus, we obtain a distributed power control algorithm which brings minimum overhead to the network for the only information the mobile needs in order to update its power level, other than its own most recent power level and the system parameters, is the level of total received power at the BS.

3.1 Stability in the Static Case

We first establish the stability of the update scheme (4) under some sufficiency conditions in the *static case*, where each mobile is connected to a specific base station (belongs to a cell) for all times . By taking the second derivative of x_i (connected to cell l) with respect to time we obtain

$$\ddot{x}_i = \left(-a_i - \frac{d^2 P_i}{dp_i^2}\right)\dot{x}_i + \sum_{j\in\mathcal{M}_l,\, j\neq i} b_i \dot{x}_j + \sum_{k\notin\mathcal{M}_l} b_i \frac{h_{kl}}{h_k}\dot{x}_k := \dot{\phi}_i(\mathbf{x}), \qquad (5)$$

where a_i and b_i are defined as

$$a_i := \left|\frac{d^2 U_i}{d\gamma_i^2}\right| \frac{L^2 \lambda_i h_i^2}{(\sum_{j\neq i}(h_{jl}/h_j)x_j + \sigma_l^2)^2},$$

and

$$b_i := \frac{a_i}{L}\gamma_i - \frac{dU_i}{d\gamma_i}\frac{L\lambda_i h_i^2}{(\sum_{j\neq i}(h_{jl}/h_j)x_j + \sigma_l^2)^2}.$$

Notice that, a_i is positive, and under *A3*, b_i is negative.

Let us introduce the candidate quadratic Lyapunov function

$$V := \sum_{i\in\mathcal{M}} \phi_i^2(\mathbf{x}) \ . \qquad (6)$$

First note that because of the uniqueness of the NE, $\mathbf{x}^*$, $\phi_i(\mathbf{x}) = 0 \ \forall i$ if and only if $\mathbf{x} = \mathbf{x}^*$. Hence, V is positive for all $\mathbf{x}$ except for $\mathbf{x} = \mathbf{x}^*$. Furthermore, as $||\mathbf{x}|| \to \infty$, $(dU_i/d\gamma_i)(L\lambda_i h_i^2)/(\sum_{j\neq i}(h_{jl}/h_j)x_j + \sigma_l^2)$ is bounded by *A2* and $|\lambda_i h_i dP_i/dp_i| \to \infty$ by *A1*. Hence, V is radially unbounded, $V \to \infty \Leftrightarrow ||\mathbf{x}|| \to \infty$, where $||.||$ denotes the norm operator.

Taking the derivative of V with respect to t we have

$$\dot{V} \leq \sum_{i\in\mathcal{M}} -2a_i\phi_i^2 + \sum_{i\in\mathcal{M}} |b_i| \sum_{j\neq i} 2\frac{h_{ji}}{h_j}|\phi_i\phi_j|,$$

where h_{ji} denotes the channel gain of mobile j to the BS to which mobile i is connected. We note that $\frac{h_{ji}}{h_j} < 1$ for all $i, j \in \mathcal{M}$, and $\frac{h_{ji}}{h_j} \ll 1$ if there is a large geographic distance between the mobiles i and j.

It follows from a simple algebraic manipulation that

$$\sum_{i\in\mathcal{M}} |b_i| \sum_{j\neq i} 2\frac{h_{ji}}{h_j}|\phi_i\phi_j| \leq 2(M_{eff}-1)\max_i |b_i| \sum_{i\in\mathcal{M}} \phi_i^2,$$

where M_{eff} is defined as the largest cluster of users who have a nonnegligible effect on each other's SIR levels through in-cell and intra-cell interference. It immediately follows that, $\max_l M_l < M_{eff} \leq M$. In practice, a possible definition of M_{eff} would be

$$M_{eff} := \max_l M_l + \sum_{k\in Neighbor(l)} M_k,$$

where $Neighbor(l)$ is defined as the set of first-tier neighbors of the cell l, due to negligible effect of mobiles in other cells, which are farther away.

Using this to bound $\dot{V}$ yields

$$\dot{V} \leq (-\min_i 2a_i + 2(M_{eff}-1)\max_i |b_i|) \sum_{i\in\mathcal{M}} \phi_i^2.$$

Next, we refine assumption *A3* as follows:

A3′. $U_i(\gamma_i)$ satisfies the inequality

$$\left|\frac{d^2U_i}{d\gamma_i^2}\right|\gamma_i < \frac{dU_i}{d\gamma_i} < (1+\gamma_i)\left|\frac{d^2U_i}{d\gamma_i^2}\right|.$$

Remark 2 *A large class of logarithmic utility functions,* $U_i = u_i log(k\gamma_i + 1)$*, where* $k > 1$ *and* $u_i > 0$ *are scalar parameters, satisfy assumptions A2 and A3′.*

Under *A3′*, we have $0 \leq |b_i| \leq a_i/L$. Hence, a sufficient condition for $\dot{V} < 0$, uniformly in the x_i's, is

$$L > m(M_{eff}-1), \tag{7}$$

where m is defined as

$$m := \max_{\mathbf{x}\in X} \frac{\max_{i\in\mathcal{M}} a_i}{\min_{i\in\mathcal{M}} a_i}. \tag{8}$$

Thus, V is indeed a Lyapunov function, and being also radially unbounded in x_i's, it readily follows that $\phi_i(\mathbf{x}(t)) = \dot{x}_i(t) \rightarrow 0,\ \forall\, i$. This in turn implies that $x_i(t)$'s converge to the unique Nash equilibrium. Hence, the unique NE point (Theorem 1) is globally asymptotically stable with respect to the update scheme (4) under the sufficient condition (7). This result is stated in the following theorem:

Theorem 3 *Let* $\mathbf{x}^{NE} := [\mathbf{x}_1^*, \mathbf{x}_2^*, \ldots, \mathbf{x}_L^*]$ *be the unique NE of a static multicell CDMA wireless network, where* $\mathbf{x}_i^* := [x_{1i}^*, x_{2i}^*, \ldots, x_{M_l i}^*]$*, and each mobile* $i \in \mathcal{M}_l$ *in cell* $l \in \mathcal{L}$ *stays connected to the respective BS for all times. Then, the system dynamics are globally asymptotically stable if*

$$L > m(M_{eff}-1),$$

where M_{eff} is defined as the largest cluster of users who have a nonnegligible effect on each other's SIR levels and m is given by (8).

3.2 The Dynamic Case and Hybrid Modeling

In the static case there are no handoffs (switches) in the network. Consider now the *dynamic case*, where mobiles connect to base stations dynamically using criteria like SIR or channel gain as they move along the cells. Then, it is possible to consider each *static case* with a fixed distribution of users among cells as a separate subsystem, q, belonging to a family (set) of systems denoted by Q. This leads to a hybrid system where each handoff corresponds to switching from one system to another. In the study of this hybrid system we make use of the concept of *dwell-time*, τ, which quantifies the minimum amount of time between two switches. However, in a wireless network handoffs are random in nature, and they may occur in short bursts. Therefore, we also make use of the concept of *average dwell-time*, which is much less restrictive than the dwell-time [13]. Let us denote the number of discontinuities of a switching signal σ on an interval (t, T) by $N_\sigma(t, T)$. Using the definition in [14], σ has average dwell time $\tau_a >$ if there exists a positive integer N_0 such that

$$N_\sigma(t, T) \leq N_0 + \frac{T-t}{\tau_a},\ \forall T \geq t \geq 0.$$

Based on our previous analysis for the static case, we define a quadratic Lyapunov function $V^{(q)}$ as in (6) for the subsystem $q \in Q$. Modifying condition (7) as

$$L > m(M_{eff} - 1) + \epsilon \tag{9}$$

where $\epsilon > 0$ is a positive constant yields

$$\dot{V}^{(q)} \leq -\epsilon V^{(q)}.$$

The unique NE, $\mathbf{x}_q^{NE}$, of the q^{th} subsystem is then globally exponentially stable by Theorem 3.

To simplify the rest of the analysis, we will make the following assumption without any loss of generality:

A5. In the multicell wireless network, no two handoffs can occur at the same time.

Since, under *A5*, the times of occurence of multiple handoffs may still be arbitrarily close to each other, this technical assumption is not restrictive in practice. As a result of *A5*, switching can happen only between "neighbor" subsystems, due to the handoff of a single mobile between two neighboring cells in the network. Hence, there exists a finite $\mu > 1$ such that

$$\frac{V^{(q)}}{V^{(r)}} \le \mu,$$

where q, $r \in Q$ are any two "neighbor" subsystems. Let us also define the set of NE for all subsystems as

$$\mathcal{N} := \{\mathbf{x}_q^{NE}, \forall q \in Q\}.$$

At a given time instance the system has only one NE which is an element of the set $\mathcal{N}$. However, this unique NE also switches from one equilibrium to the next in the set $\mathcal{N}$ with each handoff in the system.

We now extend the results of Theorem 3.2 of [14] (Theorem 4 of [13]), to multiple equilibrium points. Notice that, Theorem 3.2 of [14] does not apply directly in our case as the equilibrium point of the multicell system shifts with each switching.

Theorem 4 *Consider a family of systems, Q, defined by $\dot{x} = f_q(x)$, $\forall q \in Q$ with x_q^* being the unique equilibrium of the q^{th} system. Suppose that there exist $\mathcal{C}^1$ functions $V^{(q)} : \mathbb{R}^n \to \mathbb{R}$, $q \in Q$, class $\mathcal{K}_\infty$ functions $\alpha_1^{(q)}$ and $\alpha_2^{(q)}$, and a positive number ϵ such that we have*

$$\alpha_1^{(q)}(||x - x_q^*||) \le V^{(q)}(x) \le \alpha_2^{(q)}(||x - x_q^*||),\ \forall q \in Q, \tag{10}$$

and

$$\dot{V}^{(q)}(x) \le -\epsilon V^{(q)}(x),\ \forall x. \tag{11}$$

Suppose also that there exists a finite $\mu > 1$ such that

$$\frac{V^{(q)}}{V^{(r)}} \le \mu, \quad q, r \in Q. \tag{12}$$

Let $\overline{\mathcal{N}}$ be the union of the smallest level sets of $V^{(q)}$, $\forall q \in Q$ that contain the set of equilibria $\mathcal{N} := \{\mathbf{x}_q^*, \forall q \in Q\}$. Then, the switched system globally asymptotically converges to the set $\overline{\mathcal{N}}$ for every switching signal σ with average dwell-time

$$\tau_a > \frac{\log \mu}{\epsilon}. \tag{13}$$

Furthermore, $\overline{\mathcal{N}}$ is invariant under the same set of conditions if the dwell-time τ satisfies

$$\tau > \frac{\log \mu}{\epsilon},\ \forall t. \tag{14}$$

PROOF. Let us consider the time interval $(0, T)$ with switching times $t_1, \ldots, t_S$ where we define $T > 0$, $t_0 := 0$, and $S := T/\tau_a$ without loss of generality. Given the switching signal σ, define the piecewise differentiable function $W(t)$ as

$$W(t) := e^{\epsilon t} V^{(\sigma)}.$$

On each interval $[t_i, t_{i+1})$ (between switching times), we have from (11),

$$\dot{W}(t) \leq \epsilon W(t) - e^{\epsilon t} V^{(\sigma)} \leq 0.$$

By (12), this implies that

$$W(t_{i+1}) \leq \mu e^{\epsilon t_i} V^{(\sigma)}(x(t_i)) \leq \mu W(t_i).$$

Repeating this for all $i = 0, \ldots, S-1$, and using the definition of $W(t)$ we obtain

$$V^{(\sigma)}(x(T^-)) \leq e^{-\epsilon T} \mu^S V^{(\sigma)}(x(0)).$$

Then, from (13) and definition of S we have

$$V^{(\sigma)}(x(T^-)) \leq e^{(\frac{\log \mu}{\tau_a} - \epsilon)T} V^{(\sigma)}(x(0)). \tag{15}$$

It directly follows that $V^{(q)}$ decreases and x converges to x_q^* at time T^- for some system $q \in Q$ as T increases. Thus, by the definition of $\overline{\mathcal{N}}$ the system converges globally asymptotically to $\overline{\mathcal{N}}$. Notice that, since at the time of switching, T, the system equilibrium shifts from one point to another, no point in the system can be asymptotically stable.

We now show the invariance of $\overline{\mathcal{N}}$ under (14). Suppose that $\overline{\mathcal{N}}$ is not invariant. Then, there exists a trajectory that starts at a point $\mathbf{x} \in \overline{\mathcal{N}}$ and leaves this set. This, however, would correspond to an increase in $V^{(q)}$ for some $q \in Q$, which leads to a contradiction by (14) and (15). Hence, $\overline{\mathcal{N}}$ is invariant under the set of conditions of the theorem. $\square$

Based on Theorem 4 we next give the following result on the multicell wireless network:

Theorem 5 *Assume that the following condition holds for all the cells in the wireless network, for some* $\epsilon > 0$*:*

$$L > m(M_{eff} - 1) + \epsilon \ , \forall l \in \mathcal{L},$$

where

$$m := \max_{\mathbf{x} \in X} \frac{\max_{i \in \mathcal{M}} a_i}{\min_{i \in \mathcal{M}} a_i}.$$

Let $\overline{\mathcal{N}}$ *be the union of the smallest level sets of* $V^{(q)}(x)$ *that contain the Nash equilibria, which are "neighbor" to* q*,* $\forall q \in Q$*. Then, under the set of assumptions A1, A2, A3′, A4, A5, the dynamics of the multicell power control game globally exponentially converge to* $\overline{\mathcal{N}}$ *for every switching signal* σ *with average dwell-time*

$$\tau_a > \frac{\log \mu}{\epsilon}.$$

Furthermore, $\overline{\mathcal{N}}$ is invariant under the same set of conditions if the dwell-time τ satisfies

$$\tau > \frac{\log \mu}{\epsilon}, \ \forall t.$$

PROOF. The conditions (10) and (11) in Theorem 4 follow directly from (9) and from the properties of V under the assumptions *A1, A2, A3′, A4, A5*. We note that, due to the assumption *A5*, $\overline{\mathcal{N}}$ is a smaller set in this special case than the most general one. The rest of the proof follows the same lines as the ones in Theorem 4. □

Remark 6 *By assumption A5, the equilibrium point of the system jumps from NE of a subsystem to the NE of a "neighbor" subsystem which is close in "distance" to it, where the distance is defined by a chosen norm. Hence, in a wireless network where distribution of mobiles changes slowly over time, the operating point of the system may stay in a subset of $\mathcal{N}$ over a time interval much longer than the update interval. In this case, the practical "size" of the set $\overline{\mathcal{N}}$ is much smaller than the one in Theorem 5.*

Proposition 7 *Consider a wireless network with M mobiles and $\bar{L}$ non-empty cells. If $M \to \infty$ then $\mu \to 1$, and hence, $\tau \to 0$.*

PROOF. Without loss of generality, assume that the k^{th} mobile switches from a cell $m \in \mathcal{L}$ to cell $n \in \mathcal{L}$. Then, by definition of μ, and showing explicitly its dependence on M,

$$\mu^{(M)} = \frac{\Xi_{m,n} + \sum_{i \in \mathcal{M}_m} \phi_i^2 + \sum_{i \in \mathcal{M}_n} \phi_i^2}{\Xi_{m,n} + \sum_{i \in \mathcal{M}_m, i \neq k} \bar{\phi}_i^2 + \sum_{i \in \mathcal{M}_n} \bar{\phi}_i^2 + \phi_k^2},$$

where $\Xi_{m,n} = \sum_{l \in \mathcal{L}, l \neq m,n} \sum_{i \in \mathcal{M}_l} \phi_i^2$. As $M \to \infty$,

$$\Xi_{m,n} \gg \sum_{i \in \mathcal{M}_m} \phi_i^2 + \sum_{i \in \mathcal{M}_n} \phi_i^2 \text{ and } \bar{\phi}_i^2 \to \phi_i^2, \ \forall i \in \mathcal{M}_n, \mathcal{M}_m.$$

Hence, $\mu^{(M)}$ converges asymptotically to 1. Therefore, by Theorem 5 average dwell-time, τ, diminishes to zero. □

3.3 Stability under Feedback Delays

Even if we assume that mobiles have perfect information on their channel gain, cost, and system parameters, they still need the total received power level to be provided by the base station in order to implement the dynamic update algorithm (4).

The BS determines the total received power level through measurements, and sends it to the mobile. Measuring, processing (both at BS and mobile), and signaling of this information takes time, which results in non-negligible delays in the network, which can go up to 0.5 seconds in GSM systems (see [7], p.28). Here we model such delays as a single fixed feedback delay. Since propagation delays are negligible for cellular wireless networks, all mobiles in a cell experience almost the same amount of feedback delay. In other words, delays are symmetric within a cell. Hence, the power update function of the i^{th} mobile in terms of x_i becomes

$$\dot{x}_i(t) = \frac{dU_i}{d\gamma_i} \frac{L\lambda_i h_i^2}{\sum_{j \neq i,\, j \in \mathcal{M}_l} x_j(t-r) + \sigma_l^2} - \lambda_i h_i \frac{dP_i}{dp_i}(t) := \phi_i(\mathbf{x}(t)), \tag{16}$$

where $r > 0$ denotes the feedback delay in the network. [1]

We now investigate stability of a single cell, l, by introducing the radially unbounded, quadratic Lyapunov function

$$V_l(\mathbf{x}(t)) := \sum_{i \in \mathcal{M}_l} \phi_i^2(\mathbf{x}(t)) + (\max_{\mathbf{x} \in X_l} \max_i |b_i|)(M_l - 1) \sum_{i \in \mathcal{M}_l} \int_{t-r}^{t} \phi_i^2(\mathbf{x}(s)) ds.$$

Assuming *A1, A2, A3'*, and *A4* to hold, we essentially repeat the Lyapunov analysis of Section 3.1. Taking the derivative of $V_l(\mathbf{x}(t))$ with respect to t, we have

$$\begin{aligned} \dot{V}_l(\mathbf{x}(t)) \le & \textstyle\sum_{i \in \mathcal{M}_l} -2a_i \phi_i^2(\mathbf{x}(t)) + (\max_{\mathbf{x} \in X_l} \max_i |b_i|) \\ & \cdot \Bigg[\textstyle\sum_{i \in \mathcal{M}_l} \sum_{j \in \mathcal{M}_l,\, j \neq i} 2|\phi_i(\mathbf{x}(t)) \phi_j(\mathbf{x}(t-r))| \\ & \quad + (M_l - 1) \Big(\phi_i(\mathbf{x}(t))^2 - \phi_i(\mathbf{x}(t-r))^2 \Big) \Bigg] \end{aligned}$$

It again follows from a simple algebraic manipulation that

$$\sum_{i \in \mathcal{M}_l} \sum_{j \in \mathcal{M}_l,\, j \neq i} 2|\phi_i(\mathbf{x}(t)) \phi_j(\mathbf{x}(t-r))| \le (M_l - 1) \sum_{i \in \mathcal{M}_l} \phi_i^2(\mathbf{x}(t)) + \phi_i^2(\mathbf{x}(t-r)).$$

Using this to bound V_l further above yields

$$\dot{V}_l(\mathbf{x}(t)) \le (-\min_i 2a_i + 2(M_l - 1)) \max_{\mathbf{x} \in X_l} \max_i |b_i| \sum_{i \in \mathcal{M}_l} \phi_i^2(t)$$

Hence, a sufficient condition for $\dot{V}_l(\mathbf{x}(t)) < 0$, uniformly in the x_i's, is

$$L > m_l(M_l - 1), \tag{17}$$

[1] Here, to avoid cumbersome notational complexity we ignore the interference from neighboring cells; for extensions to multi-cell environments, see Remark 8 later.

where m_l is defined as

$$m_l := \frac{\max_{\mathbf{x} \in X_l} \max_{i \in \mathcal{M}_l} a_i}{\min_{\mathbf{x} \in X_l} \min_{i \in \mathcal{M}_l} a_i}.$$

Thus, the unique NE point (Theorem 1) of cell l is globally asymptotically stable with respect to the update scheme (16) under the sufficient condition (17) for any feedback delay r. Note that the condition in (17) is more restrictive than the one in (7). Moreover, although the value of r does not affect stability, large delays may result in slower convergence rates, and they may decrease the robustness of the system.

Remark 8 *The analysis above can be extended in a natural way to the static and dynamic multiple cell cases to obtain counterparts of Theorems 3 and 5 for the delayed information case; we have not carried out this extension here in order to keep the basic message clear.*

3.4 Communication Constraints

Total received power level, $\sum_{i \in \mathcal{M}} h_{il} p_i + \sigma_l^2$, at the BS of the cell l constitutes the main information flow in the distributed power update scheme (4). BS has to send mobiles this quantity (state information) as frequently as possible in order for the update algorithm to converge. This, however, may bring a significant overhead to the system, if not implemented efficiently. We investigate here a simple practical scheme which lessens the communication overhead, and hence, increases the efficiency through quantization (see Figure 2).

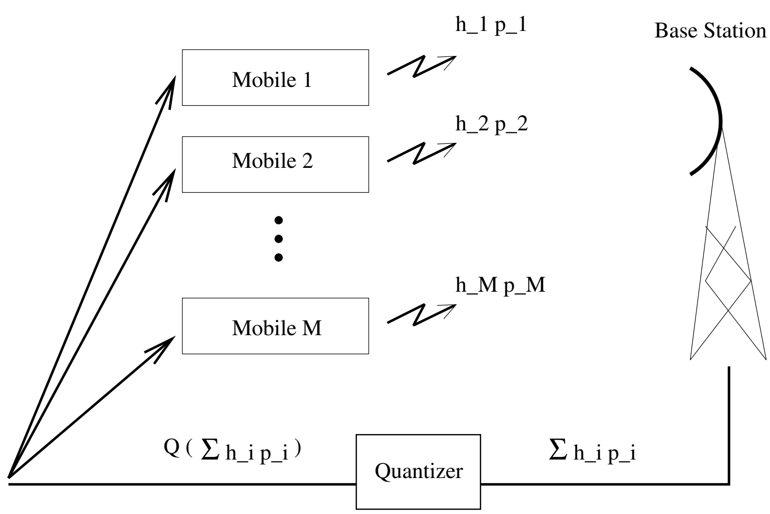

Fig. 2. A simple quantization scheme for reducing overhead in the system.

Although total received power level can be measured to a great accuracy at the BS, it is not necessary to send this information to the mobile in its most accurate form as this would waste valuable bandwidth. Instead, this value can be quantized without destroying system stability. We consider, for its simplicity and ease of implementation, a uniform quantization scheme. Assume that there exists a fixed practical upper bound on $\sum_{i \in \mathcal{M}} h_{il} p_i + \sigma_l^2$, defined as F for any cell l. Then, a K level quantization of aggregate received power level is

$$0 \leq \theta_K(\sum_{i \in \mathcal{M}} h_{il} p_i + \sigma_l^2) \leq F, \forall l \in L$$

where θ_K is a K level uniform quantizer. As long as $\sum_{i \in \mathcal{M}} h_{il} p_i + \sigma_l^2 \in [0, F]$ holds for all cells, the maximum quantization error ξ is defined as $\xi := F/2K$.

A derivation similar to the one in Section 3.1 results in the following modified version of the sufficient condition in (17) for stability of the system in the presence of the quantization error ξ,

$$L > \overline{m}(M_{eff} - 1),$$

where $\overline{m}$ is defined as

$$\overline{m} := \frac{\max_i \left| \frac{d^2 U_i}{d\gamma_i^2} \right| \frac{L^2 \lambda_i h_i^2}{(\sum_{j \neq i} (h_{ji}/h_j) x_j + \sigma_l^2)^2 + \xi}}{\min_i \left| \frac{d^2 U_i}{d\gamma_i^2} \right| \frac{L^2 \lambda_i h_i^2}{(\sum_{j \neq i} (h_{ji}/h_j) x_j + \sigma_l^2)^2 - \xi}}.$$

Hence, given a sufficiently large L, there exists an upper-bound on ξ which preserves stability. Using this value of ξ we obtain the minimum number of bits to represent the feedback information,

$$bit_{min} = \lceil \log_2 F/(2\xi) \rceil,$$

where $\lceil . \rceil$ denotes ceiling function (rounding up to the next integer). Assume that the mobile update frequency is $f_{update} Hertz$. Then, the system overhead in the downlink for a mobile is given by

$$bitrate = f_{update} \lceil \log_2 F/(2\xi) \rceil \; bps \text{ (bits-per-second).}$$

The rate of change in the total received power level in a cell can be bounded above by F_{var},

$$\frac{\partial(\sum_{i \in \mathcal{M}} h_{il} p_i + \sigma_l^2)}{\partial t} \leq F_{var}.$$

Therefore, sending the mobiles the incremental changes in the total received power level instead of sending the whole information each time results in bandwidth savings. In this case, in order to maintain the given maximum quantization error, ξ, the minimum number of bits to use is

$$\overline{bit}_{min} = \lceil \log_2 \frac{F_{var}}{2\xi f_{update}} \rceil,$$

and the bit rate is given by

$$\overline{bitrate} = f_{update} \lceil \log_2 \frac{F_{var}}{2\xi f_{update}} \rceil \; bps.$$

We note that this incremental delivery of feedback information results in significant savings of overhead bandwidth.

4 Simulations

We simulated the power control scheme developed in MATLAB. The cost function for the i-th user (mobile) was chosen as

$$J_i(x_i, \mathbf{x}_{-i}, h_i) = \frac{1}{2}\alpha_i x_i^2 - u_i \log(\gamma_i + 1), \tag{18}$$

where $\alpha_i > 0$ and $u_i > 0$ are user specific pricing and utility parameters respectively. Notice that the quadratic pricing and logarithmic utility functions in (18) satisfy assumptions *A1, A2, A3′, A4* with an appropriate choice of parameter values. Therefore, results of Theorem 5 apply to the following power update algorithm of the i-th mobile, which is connected to BS l,

$$\dot{p}_i = \lambda_i \frac{u_i}{p_i + \frac{1}{L h_i}(\sum_{j \neq i} h_{jl} p_j + \sigma_l^2)} - \lambda_i \alpha_i h_i p_i,$$

if *A5* and condition on average dwell-time are satisfied. In the simulations, a discretized version of this update scheme was implemented:

$$p_i^{(n+1)} = p_i^{(n)} + \frac{u_i}{p_i^{(n)} + \frac{1}{L h_i}(\sum_{j \neq i} h_{jl} p_j^{(n)} + \sigma_l^2)} - \alpha_i h_i p_i^{(n)}.$$

The scenario we adopted is the following. We have a simple multicell wireless network consisting of six rectangular shaped cells with 40 users. Base stations are located near the center of each cell. The system parameters are chosen as $L = 128$ and $\sigma_l^2 = 1 \; \forall l$. Cost parameters are the same for all users, $u_i = 100, \; \alpha_i = 1, \; \lambda_i = 1 \; \forall i$, and they are fixed for the duration of the simulation. Mobiles are initially located randomly in the system, and their movement is modeled as a random walk with a speed of 0.0001 units per update, where we set the update frequency to $100 \; Hz$. Hence, if we assume a system with unit size of $1000\, m$, then mobiles move with a speed of $10\, m/s$.

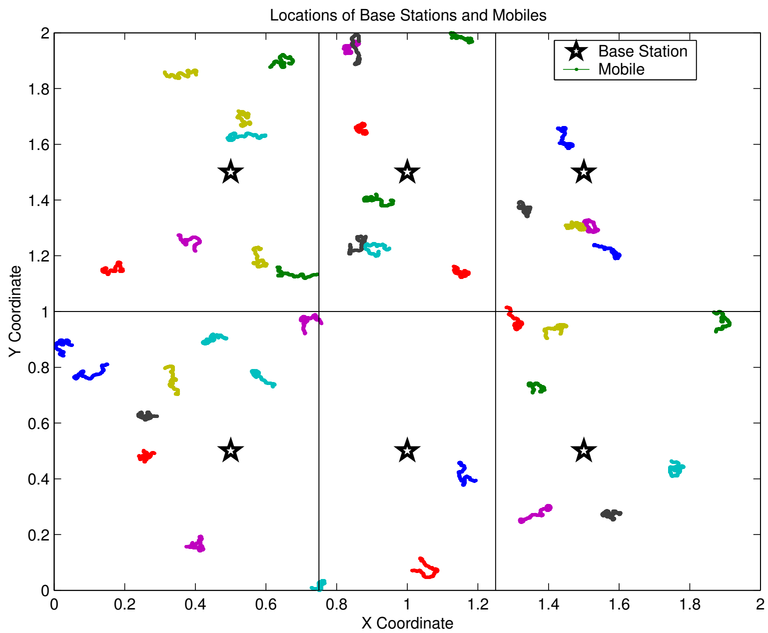

Fig. 3. Locations of base stations and the paths of mobiles.

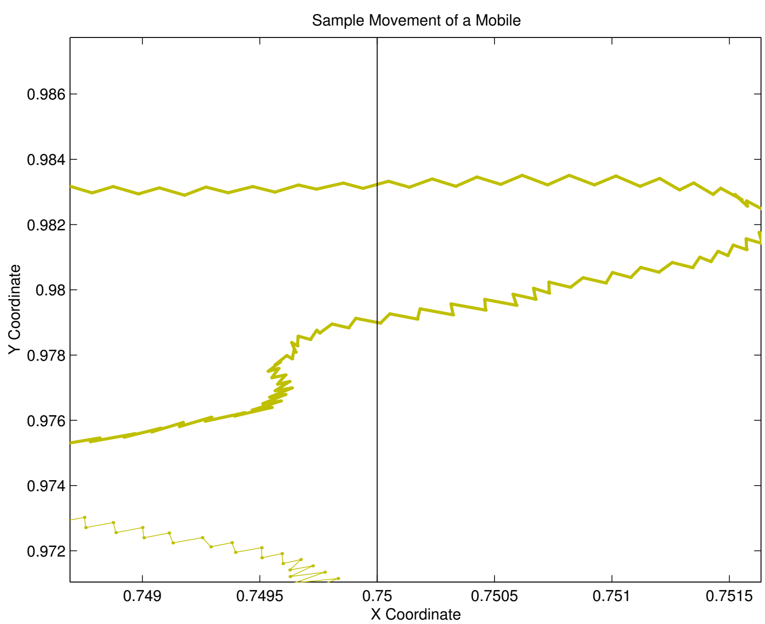

Fig. 4. A sample path of a mobile.

The channel gain of the i-th mobile is determined by a simple large-scale path loss formula $h_i = (0.1/d_i)^2$ where d_i denotes the distance to the BS, and the path loss exponent is chosen as 2 corresponding to open air path loss. The channel gain h_i is chosen as one if $d_i < 0.1$. However, fast and random movement of mobiles result in higher variations in the channel gains, and hence, compensate for this simplification. We use the channel gain as the handoff criterion. Each mobile connects to the base station with highest channel gain, which in turn corresponds to the nearest one. In Figure 3, locations of base stations and the paths of all mobiles in the network are shown. A sample path of a single mobile is shown in Figure 4.

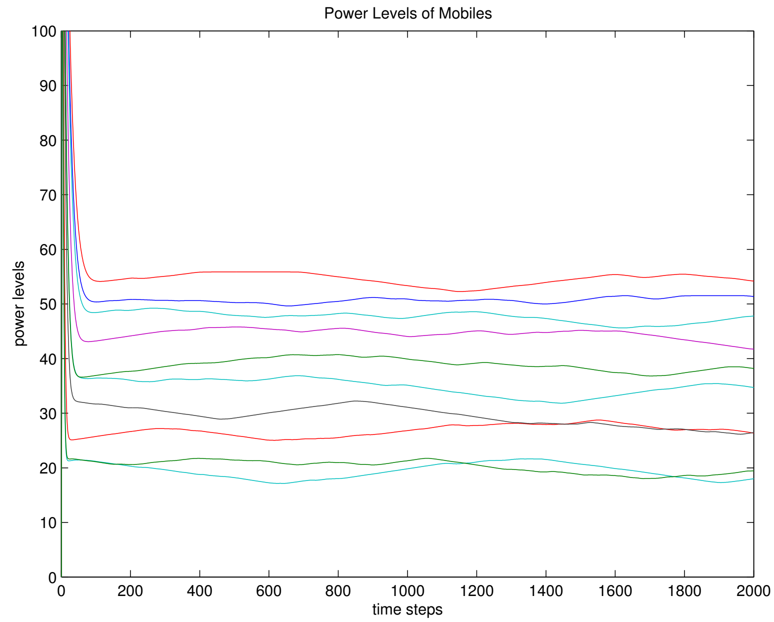

Fig. 5. Power levels of 10 selected mobiles with respect to time.

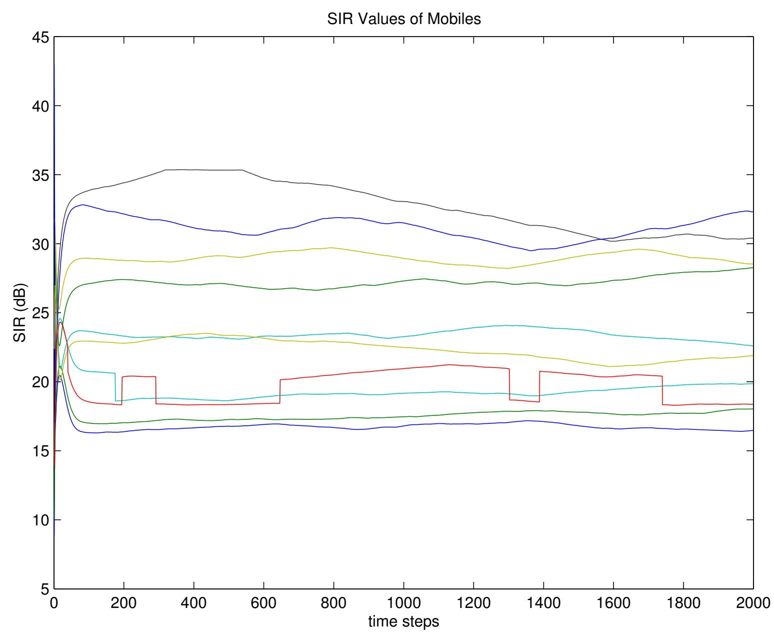

Fig. 6. SIR values of 10 selected mobiles (in dB) with respect to time.

Figures 5 and 6 depict the power levels and SIR values of mobiles for the duration of the simulation. Notice that the power levels converge to the equilibrium points, which shift due to handoffs in the system. Jumps in SIR values can be observed in Figure 6 when a mobile moves from a less congested cell with a smaller number

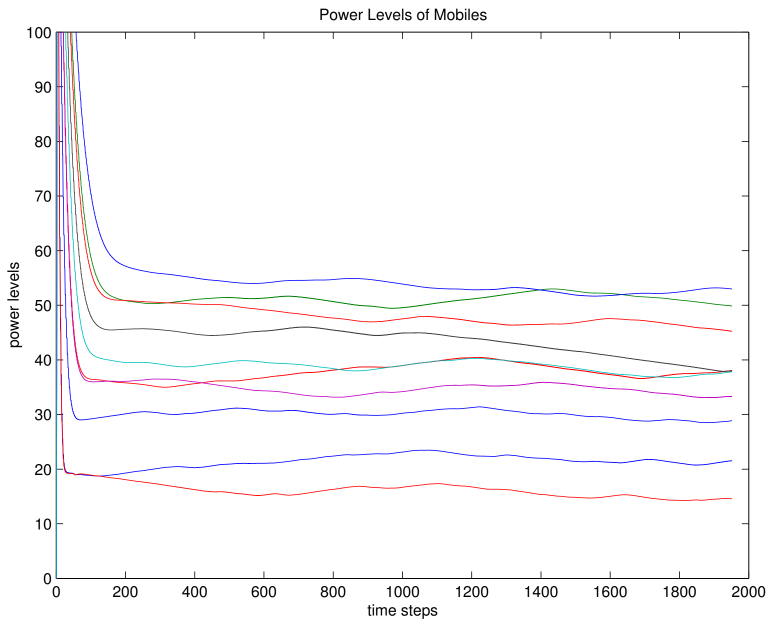

Fig. 7. Power levels of 10 selected mobiles with respect to time under a communication delay of 50 steps.

of mobiles to a more congested one or vice versa. Variations in congestion levels in the cells and in channel gains are also the reasons why not all mobiles have the same SIR levels despite having the same cost parameters. The simulation was re-

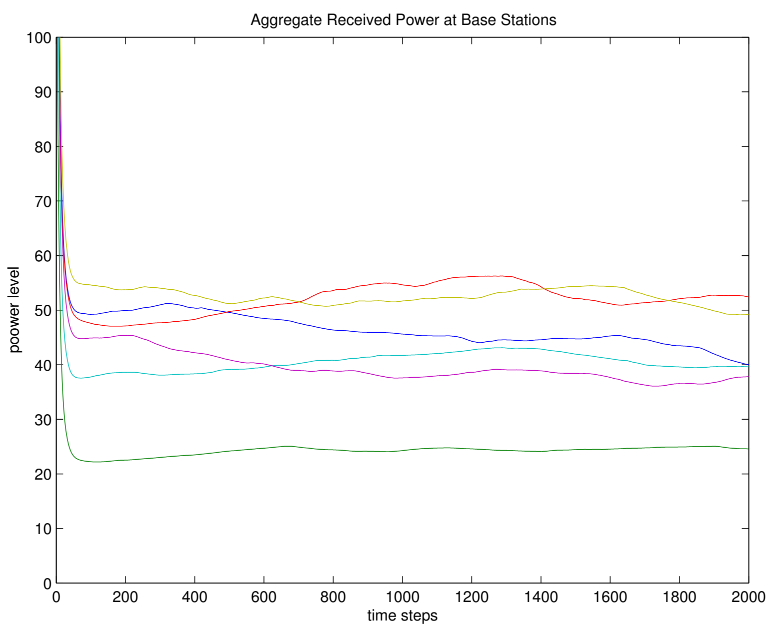

Fig. 8. Aggregate received power levels at the base stations.

peated with the same setup but with a 50 steps (0.5 seconds) communication delay between the base station and mobiles. In accordance with the results in Section 3.3, convergence characteristics of the system are not significantly affected by the presence of feedback delay except for the convergence rate, as it can be seen in Figure 7.

Figure 8 shows the aggregate received power levels at the base stations. The 'relative' smoothness of these values indicate significant savings in system overhead when the feedback quantization scheme analyzed in Section 3.4 is implemented.

5 Conclusion

In this paper, we have formulated a noncooperative power control game in a multicell CDMA wireless network, which is modeled as a switched hybrid system where handoffs of mobiles between different cells correspond to discrete switching events between different subsystems. Under a set of sufficient conditions, we have shown the existence and global exponential stability of a unique Nash equilibrium for each subsystem under a gradient algorithm. We have also established the global convergence of the dynamics of the multicell power control game to a convex superset of Nash equilibria for any switching (handoff) scheme satisfying a mild condition on average dwell-time. We have investigated the robustness of these results to communication constraints, such as feedback delays and quantization, and have presented a scheme to reduce the communication overhead between mobiles and the base stations. We have also illustrated the proposed power control scheme through MATLAB simulations.

The mathematical model developed captures a fairly broad class of convex cost functions, and addresses the multicell resource allocation problem in CDMA wireless networks. The gradient update algorithm used is market-based, distributed in nature, robust with respect to feedback delays, and requires little overhead in terms of system resources. Some directions for future research, which can be viewed as extensions of the present work, are theoretical analyses of the variations in channel gains, and the effect of discretization on the proposed update scheme.

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