Power Control for Multicell CDMA Wireless Networks: A Team Optimization Approach

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Abstract

We study power control in multicell CDMA wireless networks as a team optimization problem where each mobile attains at the minimum its individual fixed target SIR level and beyond that optimizes its transmission power level according to its individual preferences. We derive conditions under which the power control problem admits a unique feasible solution. Using a Lagrangian relaxation approach similar to [10] we obtain two decentralized dynamic power control algorithms: primal and dual power update, and establish their global stability utilizing both classical Lyapunov theory and the passivity framework [14]. We show that the robustness results of passivity studies [8, 9] as well as most of the stability and robustness analyses in the literature [10] are applicable to the power control problem considered. In addition, some of the basic principles of call admission control are investigated from the perspective of the model adopted in this paper. We illustrate the proposed power control schemes through simulations.

1 Introduction

In code division multiple access (CDMA) systems, signals of each mobile can be modeled as interfering noise for the others, leading to degradation in service. Power control in CDMA wireless networks regulates the transmission power levels of mobiles such that each user obtains a satisfactory quality of service. This goal is more precisely stated as to achieve a certain signal to interference ratio (SIR) regardless of channel conditions while minimizing the interference, and hence improving the overall performance. Minimizing the battery usage and reducing the inter-cellular interference while maintaining the desired service level are the other main objectives. The power control problem has been widely studied in the literature as one of optimization, first for voice traffic with fixed SIR requirements and then for data networks where service requirements vary from one individual user to another [1, 2, 6–8, 15]. More recently, game theoretic schemes have received the attention of the research community [1, 2, 6–8].

In this paper, we pose power control as a team optimization problem where each mobile attains at the minimum its individual fixed target SIR level, $\bar{\gamma}$, and beyond that optimizes its transmission power level according to its own preferences while respecting quality of service (QoS) constraints of other mobiles. Consequently, the power control framework considered addresses two main issues while ensuring that mobiles achieve QoS targets: it reduces both the overall interference to neighboring cells and the battery usage of mobiles. Since it would be difficult to implement any centralized power control solution, there is the need to devise decentralized power control schemes which will operate under various communication constraints limiting the flow of control information between the mobiles and the base stations. Using a Lagrangian relaxation approach similar to [10], we obtain two decentralized dynamic power control algorithms: primal and dual power update. We establish global stability of these algorithms utilizing both classical Lyapunov theory and the passivity framework [14]. One of the main contributions of this paper is to relate the power control problem defined to the existing robustness results of passivity studies [8, 9] and to most of the other stability and robustness analyses in the literature such as in [10].

The rest of this paper is organized as follows. In Sec-

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tion 2 we describe the wireless network model considered. Section 3 defines the system problem and its decomposition to user and network problems. In Section 4, we investigate a relaxation of the system problem as well as primal and dual algorithms. In addition, system dynamics and a passivity approach for stability and robustness are studied. Section 5 discusses some of the basic principles of call admission control from the perspective of the model adopted in this paper. Lastly, we illustrate the power control schemes introduced through MATLAB simulations in Section 6, which is followed by the concluding remarks of Section 7.

2 The Model

We consider a multicell CDMA wireless network model similar to the ones described in [1,2]. The wireless network consists of a set $\mathcal{L} := \{1, \ldots, K\}$ of cells, with the set of users in cell l being $\mathcal{M}_l := \{1, \ldots, M_l\}$, $l \in \mathcal{L}$, and the set of all users is defined as $\mathcal{M} := \bigcup_l \mathcal{M}_l$ with cardinality M. Users connect to the base station (BS) with the least channel attenuation within the network [1], where we make the simplifying assumption of a single BS per cell and each mobile connecting to one BS only at any given time. The i^{th} mobile transmits with a nonnegative uplink power level of $0 \leq p_i \leq p_{max}$ $\forall i$, where p_{max} is a sufficiently large upper-bound imposed for technical reasons. The received signal at the l^{th} BS, x_{il}, is the attenuated version of the transmitted power level, $x_{il} = h_{il} p_i$, where the quantity h_{il} $(0 < h_{il} < 1)$ represents the *slowly-varying* channel gain (excluding any fading). Hence, the SIR of mobile i at the base station l is given by

$$\gamma_{il} := \frac{L h_{il} p_i}{\sum_{j \in \mathcal{M},\, j \neq i} h_{jl} p_j + \sigma_l^2}.$$

Here, $L := W/R > 1$ is the spreading gain of the CDMA system, where W is the chip rate and R is the data rate of the user. Additional interference received at BS l is modeled as a fixed background noise, of variance σ_l^2. To simplify the notation we will drop subscript l if we refer to a relationship between the mobile and its own BS.

To clarify the notation adopted in this paper, we find it useful to illustrate the indexing of variables and parameters in the model with a simple example. Let us consider a wireless network of two BSs (cells) where mobiles 1 and 2 are connected to BS 1 and mobiles 3 and 4 are connected to BS 2. Then, we have $\mathcal{L} = \{1, 2\}$, $\mathcal{M}_1 = \{1, 2\}$, $\mathcal{M}_2 = \{3, 4\}$, $\mathcal{M} = \{1, 2, 3, 4\}$, and $M_1 = M_2 = 2$. Mobile 1 transmits with signal power level p_1 which is received at BS 1 as $x_{11} = h_{11} p_1$ and at BS 2 as $x_{12} = h_{12} p_1$. The SIR of Mobile 1 at its own BS 1 is denoted by γ_{11} or simply γ_1 and its SIR at BS 2 is denoted by γ_{12}. Likewise, we have $\gamma_2 = \gamma_{21}$, $\gamma_3 = \gamma_{32}$, and $\gamma_4 = \gamma_{42}$.

[1]For the simplified channel gain model considered, the BS with least channel attenuation corresponds to the nearest BS of a mobile.

3 Problem Definition

We formulate the power control problem as one of team optimization where each mobile i attains its individual fixed target SIR level, $\bar{\gamma}_i$, while optimizing its transmission power level and respecting quality of service (QoS) constraints of other users. This formulation, which we refer to as the *centralized problem*, addresses two main issues while ensuring that mobiles achieve QoS targets. First, it reduces the overall interference to neighboring cells, which is of particular importance for frequency reuse in a multicell network. Second, it reduces the battery usage of mobiles according to their individual preferences. Define $\mathbf{p} = [p_1, p_2, \ldots, p_M]$ as the vector of mobile power levels. In addition, we define $\mathbf{p}_l = [p_1, \ldots, p_{M_l}]$ as the vector of mobile power levels in cell $l \in \mathcal{L}$ and $\mathbf{x}_l = [x_{1l}, x_{2l}, \ldots, x_{M_l}]$ as the vector of received power levels at the l^{th} BS. We establish in this section that the centralized problem, $\gamma_i(\mathbf{p}) = \bar{\gamma}_i$ $\forall i \in M$, admits a unique solution, $\mathbf{p}^*$.

One of the main objectives of this paper is to study decentralized power control schemes. Therefore, we relax the centralized problem to a constrained team optimization problem, which will be referred to as the *system problem* in the remainder of the paper. Let $C_i(p_i)$ be the i^{th} mobile's individual cost function which we assume to be strictly convex and continuously differentiable. This cost function can be defined, for example, as the difference between a convex increasing cost on mobile's transmission power (battery usage) and a strictly concave utility-like function representing its willingness to increase transmission power with the aim of attaining better QoS. Then, the system problem is

$$\min_{\mathbf{p}} \sum_{i=1}^{M} C_i(p_i) \text{ such that } \gamma_i \geq \bar{\gamma}_i,\ 0 \leq p_i \leq p_{max}\ \forall i \in \mathcal{M}, \tag{1}$$

where $\bar{\gamma}_i$ is i^{th} mobile's target SIR and p_{max} is defined as a finite but sufficiently large upper-bound on p_i $\forall i$, so that the solution to (1) does not hit $p_i = p_{max}$ for any i. We note that this system problem is more general than the one in [3] which studies the special case of $C_i(p_i)$ being strictly increasing in p_i for all i.

It is convenient to rewrite the set of conditions $\gamma_i \geq \bar{\gamma}_i$, $i = 1, 2, \ldots, M$, in terms of received power levels at each

BS and in matrix form. Define the $M_l \times M$ matrix

$$\mathcal{A}_l := \begin{pmatrix} h_{1l} & -h_{2l}\dfrac{\bar{\gamma}_1}{L} & \cdots & \dfrac{-h_{M_l l}\bar{\gamma}_1}{L} \\ \dfrac{-h_{1l}\bar{\gamma}_2}{L} & h_{2l} & \cdots & \dfrac{-h_{M_l l}\bar{\gamma}_2}{L} \\ \vdots & & \ddots & \vdots \\ \dfrac{-h_{1l}\bar{\gamma}_{M_l}}{L} & \dfrac{-h_{2l}\bar{\gamma}_{M_l}}{L} & \cdots & h_{M_l l} \end{pmatrix}_{M_l \times M} \tag{2}$$

and the matrix $\mathcal{A}$ as the vertical concatenation of the $\mathcal{A}_l$, $l = 1, 2, \ldots, K$, matrices, $\mathcal{A} := \left[\mathcal{A}_1^T \cdots \mathcal{A}_K^T\right]^T$, where $[X]^T$ denotes the transpose of the matrix X. Define also the vector

$$\mathbf{b}_l := \left[\bar{\gamma}_1 \sigma_l^2 / L, \ldots, \bar{\gamma}_{M_l} \sigma_l^2 / L\right]^T,$$

and the vector $\mathbf{b} := [\mathbf{b}_1, \mathbf{b}_2, \ldots, \mathbf{b}_K]^T$. Thus, the system problem is formulated by

$$\begin{aligned} \min_{\mathbf{p}} \sum_{i=1}^{M} C_i(p_i), \quad \text{such that} \\ \mathbf{p} \in \Omega := \{\mathbf{p} \in \mathbb{R}^M : \mathcal{A}\mathbf{p} \geq \mathbf{b}, \text{and} \\ 0 \leq p_i \leq p_{max} \ \forall i \in \mathcal{M}\}. \end{aligned} \tag{3}$$

Lemma 3.1. *Let Ω be nonempty. Then, the system problem (3) is a strictly convex optimization problem with a convex, compact constraint set, and hence admits a unique global minimum.*

Proof. The objective function $\sum_{i=1}^{M} C_i(p_i)$ is clearly strictly convex in its arguments due to strict convexity of C_i with respect to p_i for all $i \in \mathcal{M}$. We now establish the convexity and compactness of the constraint set Ω. Let $\mathrm{row_i(X)}$ denote the i^{th} row of the matrix X. First, note that Ω is bounded by definition due to $0 \leq \mathbf{p} \leq p_{max}$. In addition, it is closed since it consists of intersection of half-spaces, $\mathrm{row_i}(\mathcal{A}_l)\,\mathbf{p} \geq \mathrm{b_{il}} \ \forall \mathrm{i} = 1, 2, \ldots, \mathrm{M_l}, \ \forall \mathrm{l} \in \mathcal{L}$, which include their respective separating hyperplanes, and the closed set $0 \leq \mathbf{p} \leq p_{max}$. Thus, the constraint set of the problem, Ω, is compact. Finally, following a similar argument it is easy to see that it is also convex, being the intersection of convex sets. □

We next derive a set of necessary and sufficient conditions under which Ω is nonempty and thus there exists a feasible solution to the system problem (3).

Lemma 3.2. *Let*

$$\eta_l := \sum_{j \in \mathcal{M}_l} \frac{\bar{\gamma}_j}{L + \bar{\gamma}_j}. \tag{4}$$

If $\eta_l < 1 \ \forall l \in \mathcal{L}$, then with p_{max} picked sufficiently large, Ω is nonempty. In particular, every p_l satisfying $A_l p_l \geq b_l$ also satisfies $p_l \geq 0$. If $\eta_l \geq 1$ for some $l \in \mathcal{L}$, then Ω is empty.

Proof. We first rewrite $A_l p_l \geq b_l$ as $\tilde{A}_l x_l \geq b_l$, where $x_{il} := h_{il} p_{il}$, and

$$\begin{aligned} \tilde{A}_l &= \begin{bmatrix} 1 & -\frac{\bar{\gamma}_1}{L} & \cdots & -\frac{\bar{\gamma}_1}{L} \\ -\frac{\bar{\gamma}_2}{L} & 1 & \cdots & -\frac{\bar{\gamma}_2}{L} \\ \vdots & & \ddots & \vdots \\ -\frac{\bar{\gamma}_l}{L} & -\frac{\bar{\gamma}_l}{L} & \cdots & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1+\frac{\bar{\gamma}_1}{L} & 0 & \cdots & 0 \\ 0 & 1+\frac{\bar{\gamma}_2}{L} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & 1+\frac{\bar{\gamma}_l}{L} \end{bmatrix} \\ &\quad - \begin{bmatrix} \frac{\bar{\gamma}_1}{L} & \frac{\bar{\gamma}_1}{L} & \cdots & \frac{\bar{\gamma}_1}{L} \\ \frac{\bar{\gamma}_2}{L} & \frac{\bar{\gamma}_2}{L} & \cdots & \frac{\bar{\gamma}_2}{L} \\ \vdots & & \ddots & \vdots \\ \frac{\bar{\gamma}_l}{L} & \frac{\bar{\gamma}_l}{L} & \cdots & \frac{\bar{\gamma}_l}{L} \end{bmatrix}. \end{aligned}$$

Next, dividing the ith row of this matrix by $1 + \frac{\bar{\gamma}_i}{L}$ for each $i = 1, \cdots, l$, we note that $\tilde{A}_l x_l \geq b_l$ is equivalent to

$$\hat{A}_l x_l \geq \hat{b}_l \tag{5}$$

where $\hat{b}_{il} := b_{il}/(1 + \frac{\bar{\gamma}_i}{L})$ and

$$\hat{A}_l := I - \underbrace{\begin{bmatrix} \frac{\bar{\gamma}_1}{L+\bar{\gamma}_1} & \frac{\bar{\gamma}_1}{L+\bar{\gamma}_1} & \cdots & \frac{\bar{\gamma}_1}{L+\bar{\gamma}_1} \\ \frac{\bar{\gamma}_2}{L+\bar{\gamma}_2} & \frac{\bar{\gamma}_2}{L+\bar{\gamma}_2} & \cdots & \frac{\bar{\gamma}_2}{L+\bar{\gamma}_2} \\ \vdots & & \ddots & \vdots \\ \frac{\bar{\gamma}_l}{L+\bar{\gamma}_l} & \frac{\bar{\gamma}_l}{L+\bar{\gamma}_l} & \cdots & \frac{\bar{\gamma}_l}{L+\bar{\gamma}_l} \end{bmatrix}}_{=:\, T_l}. \tag{6}$$

To show that Ω is nonempty when $\eta_l < 1$, we note that the nonnegative matrix T_l defined above has eigenvalues $\{0, \cdots, 0, \eta_l\}$, and thus, spectral radius η_l. This means that, if $\eta_l < 1$, then $\hat{A}_l = I - T_l$ is a nonsingular *M-Matrix* as defined in [4, Chapter 6]. It follows from characterization (N_{39}) in [4, Chapter 6] of nonsingular M-matrices that $\hat{A}_l x_l \geq \hat{b}_l \geq 0$ implies $x_l \geq 0$. This proves that every p_l satisfying $A_l p_l \geq b_l$ also satisfies $p_l \geq 0$, and thus, Ω is nonempty.

To prove that if $\eta_l \geq 1$ then Ω is empty, we proceed with a contradiction argument. Suppose, to the contrary, that Ω is nonempty; that is, there exists $x_l \geq 0$ such that $\hat{A}_l x_l \geq \hat{b}_l$. Because each entry of $\hat{b}_l$ is strictly positive ($\hat{b}_l \gg 0$ in the notation of [4]) this means that $x_l > 0$ and $\hat{A}_l x_l \gg 0$. From characterization (I_{28}) in [4, Chapter 6], this leads to the conclusion that $\hat{A}_l$ is a nonsingular M-matrix, which contradicts $\eta_l \geq 1$. □

We now combine the results of Lemmas 3.1 and 3.2 in the following theorem.

Theorem 3.3. *Let*

$$\sum_{j\in\mathcal{M}_l} \frac{\bar{\gamma}_j}{L+\bar{\gamma}_j} < 1 \; \forall l \in \mathcal{L}. \tag{7}$$

Then, with p_{max} picked sufficiently large, Ω is nonempty, and there exists a unique positive solution, $\mathbf{p}^*$, to the system problem (3), which has the property that $p_i^* < p_{max}$ $\forall i$.

The condition in Theorem 3.3 provides an upper bound on the achievable target SIR levels as well as for the number of mobiles in a given cell, and hence, provides a soft capacity constraint for the underlying CDMA system. Consequently, we will utilize condition (7) as a guideline for the admission control scheme based on soft constraints in Section 5.

3.1 User and Network Problems

We have already established that there exists an optimal solution, $\mathbf{p}^*$, to (3) under the conditions (7), the cost functions are strictly convex, and the constraints are linear. Then, it readily follows [5, p. 310] that there exist Lagrange multiplier vectors λ, ν, and ξ, such that $\mathbf{p}^*$ minimizes the following Lagrangian function over $\mathbb{R}^M$:

$$L(\mathbf{p},\lambda,\nu,\xi) = \textstyle\sum_{i\in M} C_i(p_i) - \lambda^T(\mathcal{A}\mathbf{p}-\mathbf{b}) + \nu^T\mathbf{p} \\ +\xi^T(\mathbf{p}-p_{max}), \tag{8}$$

and since L is differentiable,

$$\nabla_{\mathbf{p}} L(\mathbf{p}^*,\lambda,\nu,\xi) = 0. \tag{9}$$

Furthermore, since p_{max} was taken to be sufficiently large, and any feasible solution is positive under (7), both ν and ξ are zero. Moreover, the Lagrange multiplier λ is unique, since the matrix $\mathcal{A}$ is full rank (and in fact invertible).

Let us introduce the vector $\mathbf{q} := \mathcal{A}^T\lambda$. Since $\lambda^T\mathcal{A}\mathbf{p} = \mathbf{p}^T\mathcal{A}^T\lambda = \mathbf{p}^T\mathbf{q}$, the Lagrangian in (8) can be rewritten as $L(\mathbf{p},\lambda,\nu,\xi) = \sum_{i\in M}[C_i(p_i) - q_i p_i] + \lambda^T\mathbf{b}$. Then, (9) is equivalent to

$$\frac{\partial L(\mathbf{p}^*,\lambda)}{\partial p_i} = \frac{dC_i(p_i)}{dp_i} - q_i = 0, \; i = 1,2,\ldots,M.$$

A noteworthy fact is that the set of constraints $\mathcal{A}\mathbf{p} \geq \mathbf{b}$ ensure positivity of $\mathbf{p}^*$, which follows directly from Theorem 3.3.

Remark 3.4. We note that if the function $C_i(p_i)$ is chosen to be strictly increasing for all i, then the solution of the system problem, $\mathbf{p}^*$, simultaneously solves the centralized problem, $\gamma_i(\mathbf{p}^*) = \bar{\gamma}_i$ $\forall i \in M$, under (7), and is given by

$$p_i^* = \frac{\sigma_l^2 + \sum_{k\notin\mathcal{M}_l} h_{kl} p_k^*}{h_{il}\big(1 + L/\bar{\gamma}_i\big)\left[1 - \sum_{j\in M_l} \bar{\gamma}_j/(L+\bar{\gamma}_j)\right]} \;\; \forall i \in \mathcal{M},$$

where l denotes the BS to which i^{th} mobile is connected.

Having thus established the existence of a centralized solution to the system problem (3), our goal is now to find a decentralized power control algorithm that solves the system problem. As the first step in this direction, we decompose the system problem into a *User$_i$* problem for the i^{th} user and a *Network* problem by defining $r_i := p_i q_i$ $\forall i$, where $q_i = \mathrm{row_i}(\mathcal{A}^{\mathrm{T}})\lambda$ and λ is the Lagrangian multiplier vector in (8). Following the approach in [10] in the context of congestion control, we define:

$$\begin{array}{l} \mathrm{User}_i : \min_{r_i} C_i(r_i/q_i) - r_i, \; r_i \geq 0, \quad \text{and} \\ \text{Network}: \; \min_{\mathbf{p}} \sum_{i\in\mathcal{M}} -r_i \log(p_i), \; \mathbf{p} \in \Omega. \end{array} \tag{10}$$

Mobile i solves its own convex *User$_i$* problem to obtain $r_i^* = q_i\,(C_i')^{-1}(q_i)$. The Lagrangian for the network problem (which is another convex optimization problem on Ω) is given by $\tilde{L}(\mathbf{p},\mu) = \sum_{i\in M} -r_i\log(p_i) + \mu^T(\mathcal{A}\mathbf{p} - \mathbf{b})$. Using the same reasoning as in the system problem, $\partial\tilde{L}(\mathbf{p}^*,\mu)/\partial p_i = 0$, $\forall i$, which in turn leads to $q = \mathcal{A}^T\mu$. Using the full rank property of $\mathcal{A}$, it follows from the definition of $\mathbf{q}$ that $\mu = \lambda$. From [10] and Theorem 2 in [11] it is known that there always exist vectors $\mathbf{p}$, $\mathbf{r}$, and $\mathbf{q}$ satisfying $p_i = r_i/q_i$ $\forall i$, such that r_i solves the User$_i$ and $\mathbf{p}$ solves the Network problems. Thus, the vector $\mathbf{p}$ is the unique solution of the original system problem.

4 A Relaxation of the System Problem and System Dynamics

Although the User and the Network problems (10) are tractable, it would be difficult to implement a solution in any centralized manner [10]. Furthermore, there is the need to devise a decentralized power control scheme which will operate under various communication constraints limiting the flow of control information between the mobiles and the BSs. In order to circumvent these difficulties we consider a relaxation of the system problem (3) by defining appropriate penalty functions. Then, we study two distributed algorithms (primal and dual) and show that they converge to the unique solution of the relaxed system problem, which is arbitrarily close to the solution of the original problem.

Let us define the penalty or barrier function, ρ_i, corresponding to the constraint $\mathrm{row_i}(\mathcal{A})\mathbf{p} \geq \mathbf{b}_\mathrm{i}$ as

$$\rho_i(\xi) := f(\mathbf{b}_i - \xi_i), \tag{11}$$

where the scalar function $f(\theta)$ is nondecreasing and continuous in its argument θ, and attains the value 0 if $\theta < 0$. We note that an alternative penalty function $\tilde{\rho}_i$ can be defined in terms of SIR levels $\tilde{\rho}_i(\mathbf{p}) := f(\bar{\gamma}_i - \gamma_i(\mathbf{p}))$. The objective function based on penalty functions ρ_i $\forall i \in \mathcal{M}$ is defined as

$$V(\mathbf{p}) := \sum_{i\in\mathcal{M}} C_i(p_i) - \int_{b_i}^{\mathrm{row_i}(\mathcal{A})\mathbf{p}} \rho_i(\xi)d\xi. \tag{12}$$

It is straightforward to show that $V(\mathbf{p})$ is strictly convex in p_i for all i. We next define the relaxed system problem

$$\min_{\mathbf{p}>0} V(\mathbf{p}). \tag{13}$$

By an appropriate choice of penalty functions, the unique solution of (13) approximates the solution of the system problem (3) arbitrarily close, which is strictly positive under the conditions of Theorem 3.3. For example, let the function $f(\theta)$ in (11) be defined as $f(\theta) := [\theta+\varepsilon]^+/\varepsilon^2$, where the function $[x]^+$ maps x to zero if $x<0$. Then, as $\varepsilon \to 0$, the relaxed system problem approximates the solution of the system problem arbitrarily close [10]. For details on a similar relaxation of the system problem in a different context we refer to [13].

4.1 The Primal Power Update Algorithm

We define the primal power update algorithm as

$$\dot{p}_i = \frac{dp_i}{dt} = \kappa_i \left(-\frac{\partial C_i(p_i)}{\partial p_i} + q_i \right),\; i = 1, 2, \ldots, M \text{ and}$$

$$\mathbf{q}(\mathbf{p}) = \mathcal{A}^T \rho(\mathcal{A}\mathbf{p}), \tag{14}$$

where ρ_i is defined in (11), $\kappa_i > 0$ is the user-specific step-size constant, and $\rho(\mathcal{A}\mathbf{p})$ is defined as $\rho(\mathcal{A}\mathbf{p}) := [\rho_1(\mathrm{row}_1(\mathcal{A})\mathbf{p}), \rho_2(\mathrm{row}_2(\mathcal{A})\mathbf{p}), \ldots, \rho_{\mathrm{M}}(\mathrm{row}_{\mathrm{M}}(\mathcal{A})\mathbf{p})]$. The primal power update algorithm enables us to implement a distributed power update scheme. Here, mobiles vary their power levels as described by $\dot{p}$ in (14) while the BS calculates the vector $\mathbf{q}$ from the received power levels and mobile preferences and feeds this information back to the mobiles. Figure 1 depicts the information flow in a single cell. Other distributed schemes and further analysis on the communication constraints can be found in [1]. We present here a stability result for the primal algorithm.

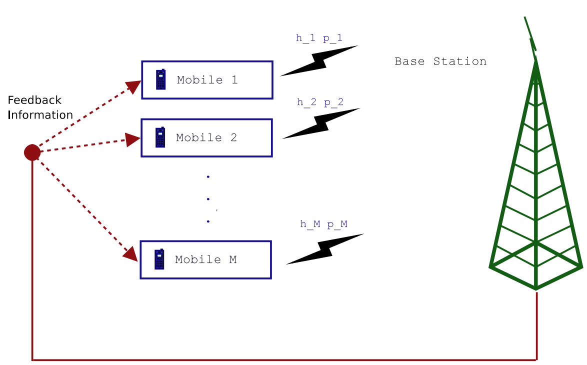

Figure 1. Information flow in a distributed power control scheme.

Theorem 4.1. *The primal power update algorithm (14) admits a unique equilibrium,* $\mathbf{p}^*$, *that solves the relaxed system problem (13). Furthermore, it is globally asymptotically stable .*

Proof. It is straightforward to see that the unique equilibrium point of the set of differential equations (14) corresponds to the unique minimum of the convex function (12):

$$\frac{\partial V(\mathbf{p})}{\partial p_i} = \frac{\partial C_i(p_i)}{\partial p_i} - q_i(\mathbf{p}) = 0$$
$$\Rightarrow p_i^* = (C_i')^{-1}(q_i(\mathbf{p}^*)).$$

Hence, the primal algorithm solves the relaxed system problem. Note by definition of ρ_i that

$$\lim_{p_i \to \infty} \sum_{i \in \mathcal{M}} \int_{b_i}^{\mathrm{row_i}(\mathcal{A})\mathbf{p}} \rho_i(\xi) d\xi \to \infty,\ \forall i \in \mathcal{M},$$

and hence,

$$\lim_{p_i \to \infty} V(\mathbf{p}) \to \infty,\ \forall i \in M.$$

Therefore, the function $V(\mathbf{p})$ is strictly convex by Lemma 3.2 in [13] and constitutes a Lyapunov function for system (14), which yields, for all $\mathbf{p} \neq \mathbf{p}^*$,

$$\dot{V}(\mathbf{p}) = \frac{\partial V(\mathbf{p})}{\partial t} = \textstyle\sum_{i=1}^M \frac{\partial V(\mathbf{p})}{\partial p_i} \cdot \dot{p}_i$$
$$= -\textstyle\sum_{i=1}^M \left(\frac{\partial C_i(p_i)}{\partial p_i} - q_i(\mathbf{p}) \right)^2 < 0.$$

Thus, the primal power update algorithm (14) is globally asymptotically stable. For an analysis parallel to this, we refer to Theorem 3.4 in [13]. □

4.2 A Passivity Approach to System Stability and Robustness

We next utilize a passivity approach to analyze stability and robustness of the primal power update algorithm. Let us study the system (14) by decomposing it into forward and feedback blocks as depicted in Figure 2, where $y := \mathcal{A}\,\mathbf{p}$. In this representation, the forward block corresponds to the mobiles and the feedback path corresponds to the base station algorithm in (14). We then have the following result:

Theorem 4.2. *Consider the feedback system (14) represented as in Figure 2. The forward system from* $(q - q^*)$ *to* $\dot{p}$, *and the return system from* $\dot{y}$ *to* $(q - q^*)$ *are both passive. Thus, the system is globally asymptotically stable.*

Proof. For the forward system, we let

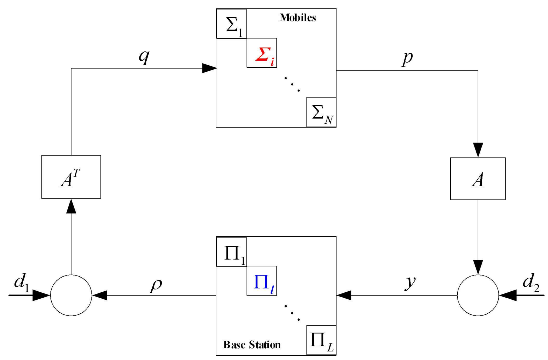

Figure 2. The primal update algorithm is represented in terms of a forward and a feedback blocks within the passivity framework.

$$V_1\left(p-p^*\right)=\sum_i\left(\left(C_i\left(p_i\right)-C_i\left(p_i^*\right)\right)-q_i^*\left(p_i-p_i^*\right)\right)$$

where $V_1(0)=0$. The derivative of each component of V_1 with respect to p_i is

$$\frac{\partial V_1}{\partial p_i}=\frac{dC_i\left(p_i\right)}{dp_i}-q_i^*$$

which, when set to zero, has the unique solution at $p=p^*$. Because the second derivative is

$$\frac{\partial^2 V_1}{\partial p_i^2}=C_1''\left(p_1\right)>0,$$

we conclude that V_1 is a positive definite function.

Next, we note that the derivative of V_1 is

$$\begin{aligned}\dot{V}_1&=\sum_i\left(\frac{dC_i\left(p_i\right)}{dp_i}-q_i^*\right)\dot{p}_i\\&=\sum_i\left(\frac{dC_i\left(p_i\right)}{dp_i}-q_i\right)\dot{p}_i+\left(q_i-q_i^*\right)\dot{p}_i\\&=\sum_i\left(\frac{dC_i\left(p_i\right)}{dp_i}-q_i\right)K\left(-\frac{dC_i\left(p_i\right)}{dp_i}+q_i\right)_{p_i}^{+}\\&\quad+\left(q_i-q_i^*\right)\dot{p}_i.\end{aligned}\qquad(15)$$

Because the first term is negative definite, as can be shown from the uniqueness of the equilibrium p^* and the discussion in Appendix C in [14], the forward system from $(q-q^*)$ to $\dot{p}$ is passive. Now consider the return system, and let

$$V_2\left(y-y^*\right)=\sum_i-\int_{y_i^*}^{y_i}\left(f\left(\xi_i\right)-f\left(y_i^*\right)\right)d\xi_i\qquad(16)$$

where $V_2(0)=0$, $\nabla V_2(0)=-\left(f(y)-f(y^*)\right)\big|_{y=y^*}=0$, and $\nabla^2 V_2=-f'(y)\geq 0$, so V_2 is a non-negative definite function. This return system from $\dot{y}$ to $(q-q^*)$ is passive since

$$\dot{V}_2=-\left(f(y)-f(y^*)\right)\dot{y}=-\left(\rho-\rho^*\right)\dot{y}.\qquad(17)$$

We can now use $V=V_1+V_2$ as a Lyapunov function and obtain

$$\dot{V}\leq\sum_i\left(\frac{dC_i\left(p_i\right)}{dp_i}-q_i\right)K\left(-\frac{dC_i\left(p_i\right)}{dp_i}+q_i\right).$$

The right-hand side is negative definite, as can be shown from the uniqueness of equilibrium p^* and the discussion in Appendix C in [14]. We thus conclude that p^* is globally asymptotically stable. □

The passivity framework enables robustness to be directly analyzed in the nonlinear system context. First, consider the disturbances d_1 and d_2 as shown in Figure 2. Using the storage functions from Theorems 1 and 2 in [9], the L_p stability (i.e., if d_1 and d_2 are L_p signals, $\left(\int_0^\infty |d_i|^p\,dt\right)^{1/p}<\infty$, then all internal signals within the loop are also L_p, $p\in[1,\infty]$) and robustness with respect to delays (A and A^T in Figure 2 replaced by delay operators) can be achieved in a similar way as in [9]. Also without losing passivity from $(q-q^*)$ to $\dot{p}$ in (15) and $\dot{y}$ to $(q-q^*)$ in (17), generalized primal algorithms can be designed and projection functions can be handled as in [14].

4.3 The Dual Power Update Algorithm

We now study the associated dual problem (8), given by

$$\max_{\mu\geq 0}\sum_{i\in\mathcal{M}}C_i\left((C_i')^{-1}(q_i(\mu))\right)-q_i(\mu)\cdot(C_i')^{-1}(q_i(\mu))+\mu^T\mathbf{b},\qquad(18)$$

where q_i is defined as $q_i:=\mathrm{row_i}(\mathcal{A}^{\mathrm{T}})\mu$. The dual power update algorithm is:

$$\begin{aligned}&\dot{\mu}_i=\kappa_i(b_i-\mathrm{row_i}(\mathcal{A})\mathbf{p}(\mu)),\ \ \mathrm{q_i}(\mu):=\mathrm{row_i}(\mathcal{A}^{\mathrm{T}})\mu,\\&p_i=(C_i')^{-1}(q_i(\mu)),\ i=1,2,\ldots,M\ .\end{aligned}\qquad(19)$$

The dual algorithm presented here is similar to the one in [13, Chapter 3] and is a special case of the dual algorithm originally presented in [10, 11]. In addition, it can be interpreted as a gradient descent algorithm to solve the dual problem (19). The next theorem is the counterpart of Theorem 4.1 and states the corresponding results for the dual algorithm.

Theorem 4.3. *The dual power update algorithm (19) admits a unique equilibrium that solves the dual system problem. Furthermore, it is globally asymptotically stable.*

Proof. It is straightforward to see that the unique equilibrium of the dual algorithm (19) coincides with the one of (18). In addition, the matrix $\mathcal{A}$ is full rank under the condition (7), and hence, given $\mathbf{q}$ there exists a unique μ such that $\mu = (\mathcal{A}^T)^{-1}\mathbf{q}$. The rest of the proof follows directly from the one of Theorem 3.5 in [13]. For the analysis of stability and robustness of the algorithm within the passivity framework, we refer to [9]. □

The dual algorithm can also be implemented as a distributed power control scheme. As in the case of the primal algorithm, the BS to which the mobile i is connected provides the term q_i, and the mobile determines its power level in accordance with (19). Hence, the algorithm (19) can be rewritten as

$$\begin{aligned} \text{BS} : \dot{\mu}_i &= \kappa_i(b_i - \mathrm{row_i}(\mathcal{A})\mathbf{p}(\mu)), \\ q_i(\mu) &:= \mathrm{row_i}(\mathcal{A}^{\mathrm{T}})\mu,\ \mathrm{i} = 1, 2, \ldots, \mathrm{M}\,, \\ \text{Mobile} : p_i &= (C_i')^{-1}(q_i(\mu)),\ i = 1, 2, \ldots, M\ , \end{aligned} \tag{20}$$

It is possible to interpret the implementation of the dual algorithm in (20) from an economic perspective. As in [10], one can consider the terms q_i as *prices* to be paid by the mobiles for creating interference to other mobiles, and hence utilizing network resources. On the other hand, the dual network problem (18) can be interpreted as one of revenue maximization from the perspective of the network operator.

Remark 4.4. In a multicell network, each BS provides feedback to (regulates) only the mobiles within its own cell and implements the relevant portion of the algorithm described in (20). Likewise, the mobiles determine their power levels by taking only the feedback from their own BSs into account. Nevertheless, our interference model takes extra-cell interference from all of the cells in the network into account in calculating the SIR levels of the mobiles. We will further investigate information flow between the BSs in Section 6 and present two schemes to implement the algorithms presented.

5 Principles for Call Admission Control

The capacity of CDMA systems is interference limited. In a CDMA system, the performance for each mobile increases (decreases) as the number of mobiles decreases (increases) [12]. In addition, the requested SIR threshold levels of mobiles, and hence their equilibrium power levels, significantly affect the capacity of a system which utilizes the power control schemes described in this paper. Therefore, a call admission control (CAC) scheme should take into account both the number of mobiles and their target SIR levels. We briefly discuss here some of the main issues for developing CAC schemes in systems implementing primal and dual power update algorithms.

We consider a local call admission control (CAC) scheme where each BS (cell) makes its call admission decisions individually according to a local criterion. In our case, a natural criterion is the set of sufficient conditions (7) for the existence of a solution to the system problem. Let δ_l denote the spare capacity in cell l for the incoming (handoff) mobiles from neighboring cells. Since interrupting an ongoing transmission is much less desirable than not admitting a call in the first place, optimization of this value is important. Define the minimum SIR level for transmission as γ_{min}. Then, from (7) and given δ_l the maximum number of incoming (handoff) mobiles from neighboring cells, which the BS l can handle, is $N_{handoff} = \lfloor \delta_l\,(1 + L/\gamma_{min}) \rfloor$, where $\lfloor \cdot \rfloor$ denotes the floor function. On the other hand, a new call requesting an SIR threshold of $\bar{\gamma}_{new}$ is admitted to cell l if the following feasibility condition is satisfied:

$$\bar{\gamma}_{new} < \frac{L}{1 - \delta_l - \sum_{j \in \mathcal{M}_l} \bar{\gamma}_j/(L + \bar{\gamma}_j)}. \tag{21}$$

We note that call blocking probability for new calls (sessions) is inversely proportional to $1 - \delta_l - \sum_{j \in \mathcal{M}_l} \bar{\gamma}_j/(L + \bar{\gamma}_j)$ in cell l. Similarly, call interruption probability is inversely proportional to the values δ_l, $l = 1, 2, \ldots, K$. All of these parameters play a significant role in designing the CAC schemes which may be posed as an optimization problem. The allocation of network resources to mobiles starting a new session and determining pricing schemes for the allocation are additional optimization issues.

6 Simulations

We simulate the primal and dual power update schemes in MATLAB. The wireless network is similar to the one in [1] and consists of 4 base stations and 10 mobiles. We consider for clarity of presentation a simple channel fading model based on large-scale path loss. Hence, the channel gain of the i^{th} mobile is determined by $h_i = (0.1/d_i)^2$ where d_i denotes the distance to the BS, and the path loss exponent is chosen as 2 corresponding to open air path loss. The system parameters are chosen as $L = 64$ and $\sigma_l^2 = 0.1$ for all cells. The mobiles are distributed randomly over the network. Figure 3 depicts the locations of the BSs and the mobiles.

In order to calculate the q vector for both the primal and the dual algorithms, each BS has to communicate with other BSs the value of the $\mathcal{A}$ matrix and μ or ρ vectors. Clearly, this "full information" case brings an additional communication overhead to the system. In order to circumvent this

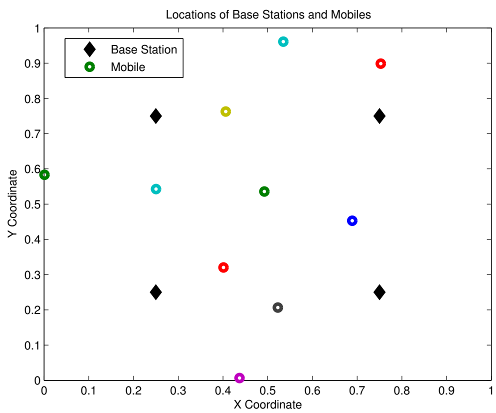

Figure 3. Locations of base stations and mobiles.

problem we propose a "low-overhead information" scheme where q is calculated at each BS using only the $\mathcal{A}_l$ matrix and the vector μ (or ρ) which contains only information on mobiles connected to that BS. Using this "approximate" information is justified due to the fact that the channel gain between the BS and other mobiles is much smaller. Hence, the remaining terms which require information exchange with neighboring BSs can be ignored in calculating the q values for mobiles in the BS. We note that better approximations can be achieved if a BS communicates with its immediate neighbors and ignores more distant ones.

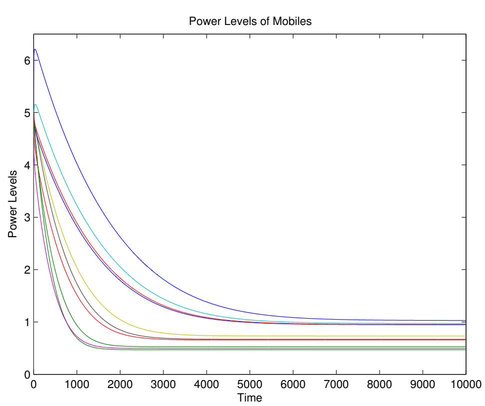

Figure 4. Power levels of mobiles with respect to time.

We first simulate the *primal power update* algorithm (14) in the full information case. The penalty function (11) for mobile i is chosen to be $\rho_i = 1000(b_i - (\mathcal{A}\mathbf{p})_i)^4$ if $b_i > (\mathcal{A}\mathbf{p})_i$ and zero otherwise.The fixed step-size of the update algorithm is $\kappa_i = 0.2$. The cost function of the i^{th}

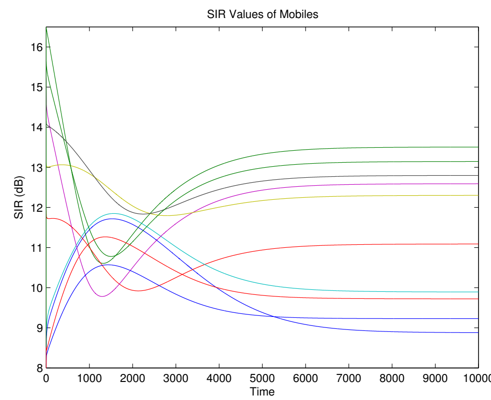

Figure 5. SIR values of mobiles (in dB) with respect to time.

mobile is the difference between its cost on battery usage (which is quadratic and proportional to the channel gain of the mobile, since distant mobiles (to the BS) need higher transmission powers) and a logarithmic function of its transmission power level: $C_i(p_i) = 5 \cdot 10^{-3} h_i p_i^2 - 10^{-4} \log(p_i)$. We arbitrarily define the target SIR levels of half of the mobiles as $10dB$ and of the other half as $7dB$. The power levels and SIR values of mobiles for the duration of the simulation are shown in Figures 4 and 5, respectively. We observe that the power levels converge to their respective equilibrium points and the SIR values are slightly higher than the target values. This is due to the logarithmic terms in the cost functions of the mobiles representing the willingness to achieve higher than target SIR levels by increasing the transmission power as long as all users reach their target SIR levels. Note that, if these additional terms are not part of the cost functions as it is the case in [3], then we observe that the final SIR values of mobiles are very close to their respective target levels.

Next, we simulate the *dual power update* algorithm (19) using both the full and low-overhead information schemes. Here, the cost function on the i^{th} mobile's battery usage is given by $C_i(p_i) = h_i p_i^2 - \log(p_i)$. Figures 6 and 7 depict the power and SIR values of mobiles for the full information case. We observe that the power levels here converge faster than the ones in the primal algorithm. The difference in the equilibrium power levels of dual and primal algorithms is due to the difference in the cost parameter. Finally, the dual algorithm is simulated using the low-overhead information scheme. From Figures 8 and 9 we conclude that the results are similar to those in the full information case.

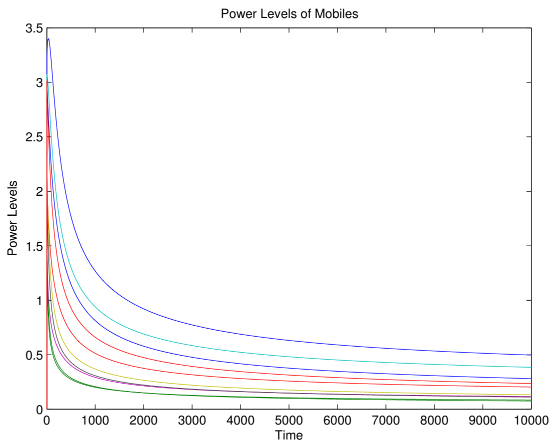

Figure 6. Power levels of mobiles with respect to time (full information).

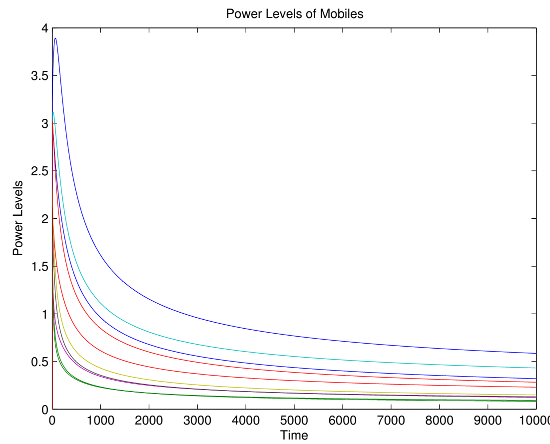

Figure 8. Power levels of mobiles with respect to time (approximate information).

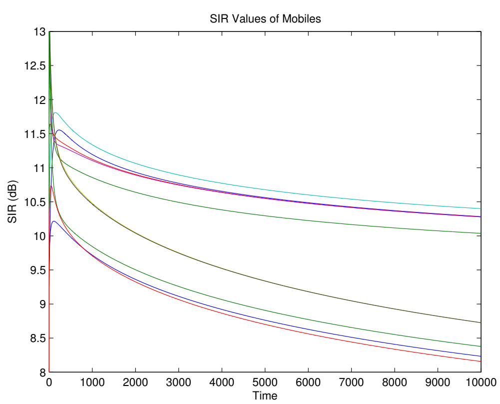

Figure 7. SIR values of mobiles (in dB) with respect to time (full information).

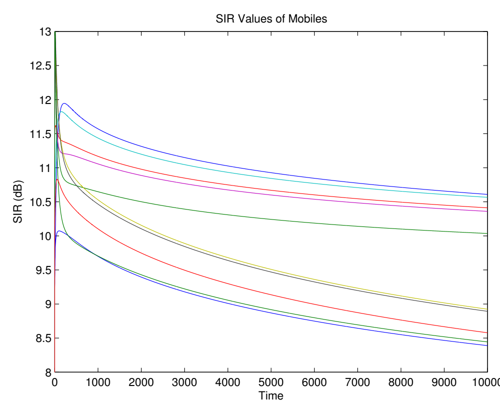

Figure 9. SIR values of mobiles (in dB) with respect to time (approximate information).

7 Conclusions

In this paper, we have defined power control in multicell CDMA wireless networks as a team optimization problem, which admits a unique optimum solution under certain conditions. Using a Lagrangian relaxation approach similar to [10, 11] we have obtained two decentralized dynamic power control algorithms: primal and dual power update, and have established their global stability utilizing both classical Lyapunov theory and the passivity framework [14]. Hence, we have shown that the robustness results of passivity studies [8,9] as well as most of the stability and robustness analyses of [10, 11] are immediately applicable to this power control problem. Furthermore, we have discussed briefly some of the basic principles of call admission control from the perspective of this paper. Finally, the power control schemes proposed are implemented and studied numerically through MATLAB simulations.

Possible extensions to this paper include formulation of the power control games in [1] and [2] as team optimization problems within the framework introduced as well as additional simulation studies to further explore the convergence and stability properties of the power control algorithms numerically.

References

[1] T. Alpcan and T. Başar. A hybrid systems model for power control in multicell wireless data networks. *Performance Evaluation*, 57(4):477–495, August 2004.

[2] T. Alpcan, T. Başar, and S. Dey. A power control game based on outage probabilities for multicell wireless data networks. In *Proc. of American Control Conference (ACC) 2004*, pages 1661–1666, Boston, MA, July 2004.

[3] T. Alpcan, X. Fan, T. Başar, M. Arcak, and J. T. Wen. Power control for multicell CDMA wireless networks: A team optimization approach. In *Proc. of the WiOpt'05 Workshop: Modeling and Optimization in Mobile, Ad Hoc and Wireless Networks*, pages 379–388, Riva del Garda, Trentino, Italy, April 2005.

[4] A. Berman and R. J. Plemmons. *Nonnegative Matrices in the Mathematical Sciences*. Classics in Applied Mathematics. Society for Industrial and Applied Mathematics, Philadelphia, PA, 1994. Originally published by Academic Press, New York, 1979.

[5] D. Bertsekas. *Nonlinear Programming*. Athena Scientific, Belmont, MA, 2nd edition, 1999.

[6] D. Falomari, N. Mandayam, and D. Goodman. A new framework for power control in wireless data networks: Games utility and pricing. In *Proc. Allerton Conference on Communication, Control, and Computing*, pages 546–555, Illinois, USA, September 1998.

[7] X. Fan, M. Arcak, and J. T. Wen. Passivation designs for CDMA uplink power control. In *Proc. of the American Control Conference (ACC) 2004*, Boston, MA, July 2004.

[8] X. Fan, M. Arcak, and J. T. Wen. Robustness of CDMA power control against disturbances and time-delays. In *Proc. of the American Control Conference (ACC) 2004*, Boston, MA, July 2004.

[9] X. Fan, M. Arcak, and J. T. Wen. Robustness of network flow control against disturbances and time-delays. *Systems and Control Letters*, 53(1):13–29, 2004.

[10] F. Kelly, A. Maulloo, and D. Tan. Rate control in communication networks: shadow prices, proportional fairness and stability. *Journal of the Operational Research Society*, 49:237–252, 1998.

[11] F. P. Kelly. Charging and rate control for elastic traffic. *European Transactions on Telecommunications*, 8:33–37, January 1997.

[12] T. S. Rapaport. *Wireless Communications: Principles and Practice*. Upper Saddle River, NJ: Prentice Hall, 1996.

[13] R. Srikant. *The Mathematics of Internet Congestion Control*. Systems & Control: Foundations & Applications. Birkhuser, Boston, MA, 2004.

[14] J. T. Wen and M. Arcak. A unifying passivity framework for network flow control. *IEEE Transactions on Automatic Control*, 49(2):162– 174, February 2004.

[15] R. D. Yates. A framework for uplink power control in cellular radio systems. *IEEE Journal on Selected Areas in Communications*, 13:1341–1347, September 1995.