A GAME THEORETICAL FRAMEWORK FOR VARIABLE RATE FLOW CONTROL AND CDMA UPLINK POWER CONTROL

BY

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ABSTRACT

For communication networks with two different scenarios, we develop two mathematical models using noncooperative game theoretical framework. The first model is for variable rate real time traffic at a bottleneck node. The second one analyzes the uplink power control problem in a CDMA system. In both models, we not only address the flow control and power control problems, but also pricing and allocation of a single resource among many users. Distributed, end-to-end flow and power control schemes are proposed by introducing a cost function, defined as the difference between pricing and utility functions in each case. Existence of a unique Nash equilibrium is proven based on the defined cost functions. In addition, three distributed update algorithms (parallel, random, and gradient update) are shown to be globally stable under reasonable conditions. The convergence properties and robustness of each algorithm are studied through extensive simulations.

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CHAPTER 1

INTRODUCTION

1.1 Background

Game theory provides a natural framework for developing pricing mechanisms to solve various problems in communication networks, such as rate control, routing, fair allocation of resources among users in wireline networks, and power control in wireless networks. In both cases, users on the network are completely noncooperative in terms of their demands for the system resources, which can be bandwidth or signal to interference ratio (SIR) depending on the network at hand. This fact motivates the use of noncooperative game theory [1] for flow, congestion, or uplink power control. An appropriate solution concept here is the noncooperative Nash equilibrium. In this approach, a noncooperative network game is defined where each user attempts to minimize a specific cost function by adjusting his flow rate or transmission power, with the remaining users' flows or powers fixed. An advantage of this approach comes from the fact that it leads to distributed schemes, and not a centralized control for the network, which fits well with today's as well as tomorrow's expected trend of decentralized computing.

In this thesis, we develop two mathematical models using the noncooperative game theoretical framework. The first model is for the variable rate real time traffic at a bottleneck node in an Internet style network. The second one analyzes the uplink power control problem in a code division multiple access (CDMA) system. In both cases we make use of the conceptual framework of game theory and also of some specific results from noncooperative games to obtain market-based control mechanisms. Problem-specific cost functions provide the pricing schemes, where supply of the resource and the demand for it lead to a unique equilibrium point. In addition, we examine three relevant distributed update schemes in terms of their convergence and robustness properties: parallel, random, and gradient update algorithms (PUA, RUA, and GUA).

1.2 Models and the Literature Overview

In the first model, we investigate the flow control problem in the Internet. The flow control mechanisms of the current Internet, implemented in TCP (Transmission Control Protocol) provide distributed, end-to-end congestion control for the Internet traffic [2]. TCP was specifically designed to provide reliable, best-effort type traffic over an unreliable internetwork. Evolution of the Internet over time, however, has resulted in a network that tries to meet very different needs, in contrast with the original design goals. Implementation of RTT (real-time traffic) on the Internet for new applications like VoIP (voice over Internet protocol) or video conferencing is one such example. Pricing of the network resources and charging the users in proportion with their usage is another challenge. The Internet is no longer the small, special community it used to be, and there is an emerging need for additional mechanisms to ensure fair allocation of network resources among the users.

Achieving these goals is only possible with new congestion and flow control mechanisms. The implementation of RTT requires certain QoS (quality of service) guarantees in a best-effort type network to ensure the necessary minimum flow rate for possible applications.

There is an ongoing effort to improve and modify the flow control mechanisms of TCP to satisfy the needs of the current Internet. Most of the literature in this area is concentrated on controlling the best-effort type traffic, where several different approaches have been used [3, 4]. These methods can be divided into three groups. The first approach is a centralized one, where the flow of each user is regulated separately by the network.

Such a centralized approach is not in line with the distributed philosophy of the Internet, which is more widely acknowledged. A second approach is to provide incentives for end users to support the continued use of end-to-end congestion control, similar to the current mechanism. This can be achieved via policing methods, where those users who do not adapt their flows in the case of a congestion are categorized as *unresponsive* and punished by the network [3]. The third approach is one of the most popular approaches, because of the fact that it addresses not only congestion control problem, but also pricing and fairness issues. The basic principle here is to provide specific pricing mechanisms to end users and to let them adjust their flow rates accordingly. The recent work of Kelly et al. [4] on shadow prices and proportional fairness is an example of this type of approach. Using a feedback mechanism based on shadow prices, one can achieve stability and optimal usage of network resources. Fairness has been defined in this work in the proportional sense, which is a relaxed form of classical max-min fairness.

Among many different approaches, the game theoretical approach has been enjoying increasing popularity as it provides a fitting framework for studying the underlying network optimization problems [5, 6, 7]. However, most network games in the literature are focused on elastic, best-effort type traffic [6]. As an example for addressing the flow control problem in a game theoretic framework, we can cite Altman et al. [5] who show that if an appropriate cost function and pricing mechanism are used, one can find an efficient Nash equilibrium for a multiuser network, which is further stable under different update algorithms. Here, we consider a model with two components. The first pertains to a classical admission control mechanism [2, p. 494], where the users are admitted to the network after given a certain QoS guarantee in accordance with the available resources of the network. The guaranteed minimum flow rate meets the requirements of the intended RTT application.

The second component of the model concerns elastic flow, and it accommodates a wide range of traffic types, from medium to high elasticity. The distributed end-to-end control system is modeled as a network game where users or players adjust their excess flow rates or strategies according to their individual needs and by taking into account the state of the network. In addition to a relevant pricing function, cost function we adopt for this purpose features an inherent feedback mechanism, enabling the users to acquire the basic (essential) information about the state of the network. The noncooperative game framework provides equilibrium conditions for the system and, most importantly, the market structure, where supply and demand for bandwidth determine the allocation of network resources and prices. Fairness is also an important issue, and this is built into the network so that those users who are willing to pay for resources more than others receive a proportionately larger portion of the resources. We model the individual user's demand for bandwidth in terms of two different utility functions: an affine and a logarithmic utility function.

In the second part of the thesis, we present a game theoretical treatment of distributed power control in CDMA wireless systems. In wireless communication systems, mobile users respond to the time varying nature of the channel, described using short and long term fading phenomena, by regulating their transmitter powers. Specifically, in a CDMA system, where signals of other users can be modeled as interfering noise signals, the major goal of this regulation is to achieve a certain SIR ratio. Hence, there are two major incentives for a user to exercise power control: The first one is the limit on the battery energy available to the mobile. The second reason is the increase in capacity, which can be achieved by minimizing the interference.

In most of the current wireless systems, the uplink power of mobile users is controlled centrally. Mechanisms like the open-loop or the closed-loop power control are implemented in CDMA systems. In the open-loop power control, the mobile regulates its transmitted power inversely proportional to the received power. In the closed-loop power control, on the other hand, commands are transmitted to the mobile over downlink to raise or lower its uplink power [8, pp. 182].

An alternative is to implement a distributed control scheme, in which each user regulates its transmitted power independently. There are several recent papers discussing the distributed power control approach. Yates [9] establishes a distributed control framework for CDMA systems, where users need to satisfy a transmitter power constraint.

This constraint can be described as the interference a user has to overcome to achieve an acceptable connection. One can come up with different interference constraints for different systems: fixed assignment, minimum power assignment, diversity, etc. Yates also shows the existence of a unique fixed point for synchronous and asynchronous power control algorithms.

Game theory is another framework that can be used for distributed power control. Possible utility functions and their properties for both voice and data users are investigated in detail in [10]. An example utility function which also takes error correction into account is formulated, and existence of a unique Nash equilibrium is shown. One interesting feature of this framework is that it provides utility functions for wireless data transmission, where power control directly affects the capacity of mobiles' data transmission rates. A linear pricing scheme is also proposed in [10] in order to achieve a Pareto improvement in the utilities of mobiles.

In an earlier study [11], Nash equilibria achieved under the pricing scheme have been characterized by using supermodularity. It has been shown that a noncooperative power control game with pricing scheme is superior to one without pricing. One deficiency of this game setup, however, is that it does not guarantee social optimality for the equilibrium points.

The structure of the model we propose for the power control game departs from the earlier ones, and is very similar to the flow control setting. We define a cost function as the difference of a linear pricing scheme proportional to transmitted power and a logarithmic, strictly concave utility function based on SIR of the mobile. Next, the existence and uniqueness of Nash equilibrium is proven for the most general case. Based on a study of Yates [9], one way to extend the model is to include certain SIR constraints. As an alternative, we suggest a pricing strategy to meet the given constraints and analyze the relation between price and SIR. In addition, different pricing strategies are investigated, and a sufficient condition is given for stability under two different relevant update schemes.

For both models, we use extensive simulations using MATLAB in order to investigate the convergence, stability and robustness of the update algorithms. Moreover, we study the effect of the various parameters of the models, especially different pricing schemes. In order to make the simulations more realistic we introduce factors like delay and disturbances by varying the number of users in the network.

The next two chapters deal with the flow control problem, with Chapter 2 describing the model adopted, the solution process, and the update algorithms, and Chapter 3 presenting simulation results. Chapters 4 and 5, on the other hand, deal with the power control problem, presenting both analytical derivation as well as results of simulation studies. The thesis ends with the concluding remarks of Chapter 6.

CHAPTER 2

THE VARIABLE RATE MODEL WITH QOS GUARANTEES

In this chapter we introduce a variable rate model with QoS guarantees for Internet traffic. The existence and uniqueness of a Nash equilibrium is shown under two different utility and cost functions. Moreover, three relevant update algorithms: parallel, random, and gradient update are considered. A sufficient condition for the convergence and stability of the update schemes is established.

2.1 The Model and the Cost Function

We consider a bottleneck node in a general topology network, with a certain level, C, of available bandwidth, which is shared by N users or connections. The i^{th} user's flow rate λ_i consists of two parts: The guaranteed minimum flow rate $\lambda_{i,min}$ and the variable excess flow rate, x_i, defined as the difference of the total flow and the minimum flow: $x_i = \lambda_i - \lambda_{i,min}$. The guaranteed flow rate, $\lambda_{i,min}$, is negotiated between the user and the network at the time of the connection setup and remains constant thereafter. A similar approach can be found in Yaiche et al. [12]. The guaranteed flow plays a crucial role in meeting the QoS requirements necessary for RTT types. The problem of giving users guarantees for their requested minimum flows while at the same time preserving the network resources, bounded by the maximum available bandwidth C at the bottleneck node, can be solved with the aid of an admission control mechanism.

The proposed admission control scheme is market-based and has a cost structure and a pricing structure. Although it can be compared with classical call blocking schemes, modeled often as $M/M/s/s$ queues in circuit switching [2], it differs in many respects and has several advantages. In this scheme, a new j^{th} user, $j = N + 1$, requesting a minimum flow rate $\lambda_{j,min}$ determines itself whether or not to initiate a session under the admission pricing function:

$$P_j^0 = \frac{k_{adm}}{C - (\lambda_{j,min} + \sum_{i=1}^{N} \lambda_i)} \tag{2.1}$$

The constant k_{adm} is determined by the network for pricing purposes. The denominator term $C - (\lambda_{j,min} + \sum_{i=1}^{N} \lambda_i)$ is responsible for setting the price of the resource, in this case the bandwidth, directly proportional to the total demand. The exchange is given the right of denying the user the requested service if the price is higher than a certain maximum threshold value. The mechanism results in a "soft" call blocking scheme, where the decision of whether or not to block a call is taken not only by the network side but also by the user, depending on the demand of the particular user for the bandwidth at that instant. At the same time, the network resources are prevented from going down to dangerously low levels in case of a congestion.

The market-based scheme has the following advantages when compared with classical admission schemes: First, the calls need not be identical in terms of their QoS requirements. Users have the freedom to choose the amount of bandwidth they request. Second, call blocking is not determined only by the network. Users may influence this decision by their demand level, reflected by the amount they are willing to pay for the connection. Finally, the market-based approach ensures a fair distribution of resources, where prices are determined purely by supply and demand.

The guaranteed flow rate $\lambda_{i,min}$, despite its necessity for real time applications, is inherently inflexible. Once the user negotiates with the network at the beginning of the connection and is admitted to the system, the terms cannot be changed during the connection. It is conceivable, however, that there might exist applications that would demand additional bandwidth during the course of the connection. In order to add this flexibility to the system, we consider here a network game, in which the i^{th} user can regulate his excess flow x_i.

We note that the excess flow rate is elastic; i.e., it has no QoS guarantees and is bounded above by the total available excess bandwidth m. This is the remaining available bandwidth after all guaranteed minimum flows are subtracted from the total capacity:

$$m = C - \sum_{i=1}^{N} \lambda_{i,min} \tag{2.2}$$

The proposed admission scheme for minimum flow rate, in spite of its differences, can be implemented similarly to classical admission schemes. Therefore, we will focus here on the excess elastic flow part of the model. The network game is defined at the bottleneck node, using a specific cost function and a totally distributed control scheme, where end users adjust their flow rates themselves. Consistent with the assumption of rationality, users minimize their costs, determined by their cost function. Overall control of the network is achieved through setting the pricing parameter and adjusting the maximum available capacity C, which does not have to be the real physical capacity. Another function of the network is that it detects and limits unresponsive flows in the case of users with malicious intentions [3]. Each user may enter the network game for excess bandwidth and minimizes its cost by regulating its excess flow rate x_i after its constant flow rate $\lambda_{i,min}$ is determined. A natural minimum for the case where the user has no demand for excess bandwidth is $x_i = 0$ or $\lambda_i = \lambda_{i,min}$. The intuitive explanation for this is that the user is completely satisfied with the prenegotiated constant flow $\lambda_{i,min}$. Then, such a user does not need to enter the network game at all. Excluding such users from the network game simplifies the analysis. As a result, the number of users in the network game M can be less than the total number of users N or $M < N$. The remaining available bandwidth m is adjusted accordingly.

The cost function for the users entering the game is defined as the difference between the pricing and the utility functions. This cost function not only sets the dynamic prices, but also captures the demand of a user for bandwidth. The first term of the cost function, pricing function, is defined as follows

$$P_i(\lambda_i) = \frac{k_i}{C - \lambda}(\lambda_i - \lambda_{i,min})^2 + l_i \lambda_{i,min} \tag{2.3}$$

Here, λ is the total flow of all users: $\lambda = \lambda_i + \lambda_{-i}$, where λ_{-i} is the sum of the flows of all users except the i^{th} one, and $k_i \geq 0$ and $l_i \geq 0$ are pricing parameters determined by the network. Notice that l_i can be considered the fixed price the user pays for the guaranteed bandwidth $\lambda_{i,min}$. The precise value of this parameter is, however, not that important, since it does not affect the optimal flow rate of the user. The pricing term not only sets the actual price, but also has the regulatory function of giving the user feedback about the network status via the denominator term $C - \lambda$. In queueing systems, this term is generally interpreted as the delay. In the present context, however, it has a feedback functionality. As the sum of flows of users approach the capacity C, the denominator approaches zero, and hence the price increases without bound. This preserves the network resources by forcing the users to decrease their elastic flows. Concurrently, a proportional relationship between demand and price is obtained, which ensures that the prices are set according to market forces.

In terms of classical queueing theory, our approach might seem to lead to counterintuitive behavior, in the sense that the users have to pay a higher price for longer delays. But the same high cost is forcing users to review their demand for the excess bandwidth and decrease their flows, if their utility from using the excess bandwidth does not meet the price they pay. The proposed mechanism achieves fairness in the sense that users receive the amount of bandwidth proportional with their demand. It is expressed via the price they are willing to pay for the resource (the bandwidth). The status of the network, total demand for bandwidth at any instant, also affects the prices and thereby the availability of the resource for the user. We wish to note that a similar but simplified structure is widely accepted and has been used in public switched telephone networks (PSTN) for years. In busy hours the users are charged higher prices than nights and weekends, although there is no difference in terms of the QoS they get, other than the difference in total demand for the resource or service.

The second part of the cost function, the utility function U_i, quantifies the user's utility for having the bandwidth and captures to some extent the "human factor." Although it cannot be exactly known to the network, some statistical estimates can be collected, taking into account habits of specific type of a user over a certain time period. A reasonable assumption is to define it as strictly concave for elastic flows. As widely used and accepted in economics, a logarithmic function can be chosen as the best approximation to the utility function of the user in this case.

In order to capture the properties of real time traffic to the fullest extent, the utility of the i^{th} user U_i is examined in two parts: The excess utility function U_i^e, defined in terms of the excess flow rate $x_i \geq 0$, quantifies the user's demand for excess bandwidth. On the other hand, the utility for the region $x_i < 0$ is described with either a strictly convex function that models the requirements of the real time traffic from the user's point of view or a zero function. In both cases, if the user's flow rate is less than the guaranteed flow rate $\lambda_{i,min}$ the general utility drops fast to zero. Having more than the minimum rate, on the other hand, might not increase the utility when the user has no demand for excess bandwidth. In this case, the utility function U_i is either a simple step function or strictly convex in the region $\lambda_i < \lambda_{i,min}$, and U_i^e is a constant. In accordance with the previous discussion, we exclude such users from the network game.

For the case where the user demands excess bandwidth, the utility function U_i is formulated in the general form, with the aid of excess utility U_i^e:

$$U_i(\lambda_i) = \begin{cases} U_i^e(x_i), & \lambda_i \geq \lambda_{i,min} \\ f(\lambda_i), & \lambda_i < \lambda_{i,min} \end{cases} \tag{2.4}$$

where $f(\lambda_i)$ is either strictly convex and increasing or zero in the limiting case. Due to the nature of RTT, $f(\lambda_i)$ is bounded above by a constant d_i, implying the utility obtained from the minimum flow $\lambda_{i,min}$. Additionally, we assume U_i to be continuous, unless $f(\lambda_i)$

is zero. As a result, we capture a broad range of utility functions for traffic types of low elasticity.

Under the assumption of logarithmic utility, a possible realistic utility function for a user demanding excess flow can be defined in terms of x_i:

$$U_i^e(x_i) = \ln(1 + x_i) + d_i \quad , x_i \geq 0 \ \ \forall i \tag{2.5}$$

Figure (2.1) shows two typical utility functions.

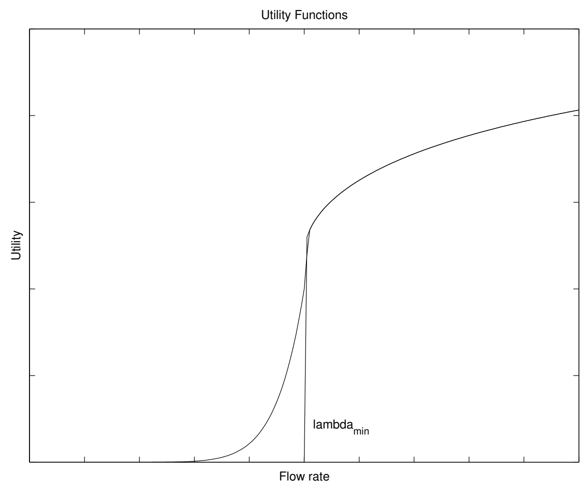


Figure 2.1 Two sample utility functions with logarithmic excess utility.

Based on the given pricing and utility functions, the cost function is simply $P - U$. In other words, the flow rate of a user results from the interaction between price and demand in terms of excess flow:

$$J_i(x_i, x_{-i}) = \begin{cases} \dfrac{k_i x_i^2}{m - (x_i + x_{-i})} - \ln(1 + x_i) + e_i, & x_i \geq 0 \\ \dfrac{k_i x_i^2}{m - (x_i + x_{-i})} + l_i \lambda_{i,min} - f(x_i + \lambda_{i,min}), & x_i < 0 \end{cases} \tag{2.6}$$

where $e_i \equiv l_i \lambda_{i,min} - d_i$ is a constant and has no effect in the optimization process.

The same cost function for the i^{th} user can also be expressed in terms of the total flow of the user, which we also denote by J_i, by a slight abuse of notation:

$$J_i(\lambda_i, \lambda_{-i}) = \begin{cases} \frac{k_i}{C-\lambda}(\lambda_i - \lambda_{i,min})^2 + l_i\lambda_{i,min} - \ln(1+\lambda_i - \lambda_{i,min}) - d_i, & \lambda_i \geq \lambda_{i,min} \\ \frac{k_i}{C-\lambda}(\lambda_i - \lambda_{i,min})^2 + l_i\lambda_{i,min} - f(\lambda_i), & \lambda_i < \lambda_{i,min} \end{cases} \tag{2.7}$$

Notice that the excess utility function is defined only in the region where $\lambda_i > \lambda_{i,min}$, and utility for the remaining part is defined by the convex function $f(\lambda_i)$. In the next section, we will revisit this point and show that a Nash equilibrium cannot occur in the region $x_i \leq 0$ for any i, even if $x_i \leq 0$ is allowed.

A drawback of the realistic utility function above is that it leads to nonlinear reaction functions for the users. Therefore, an analytical analysis of the cost function with this utility is very difficult and limited, if not impossible, even though an existence and uniqueness result (on Nash equilibria) could be obtained, as we will do in next section. In order to make the analysis tractable, however, for explicit results, we will use linear utility functions for the users, which leads to a set of linear equations as reaction functions. Accordingly, we will take as the utility function of user i:

$$U_i^e(x_i) = a_i x_i + d_i \tag{2.8}$$

where a_i is a positive constant not exceeding 1. One possible interpretation for the linear utility is that it constitutes a linear approximation to the actual utility function at any point λ_i. In this case, the system is analyzed locally in the vicinity of the chosen point $\lambda_i = x_i + \lambda_{i,min}$. The parameter a_i is the slope of the utility function at that point:

$$a_i \equiv \frac{\partial U_i^e(x_i)}{\partial x_i} \Rightarrow a_i \leq 1 \quad , \forall i \tag{2.9}$$

The cost function (2.7) based on the linear utility function (2.8) is given by:

$$J_i(x_i, x_{-i}) = \begin{cases} \frac{k_i x_i^2}{m-(x_i + x_{-i})} - a_i x_i + \tilde{e}_i, & x_i \geq 0 \\ \frac{k_i x_i^2}{m-(x_i + x_{-i})} + l_i\lambda_{i,min} - f(x_i + \lambda_{i,min}), & x_i < 0 \end{cases} \tag{2.10}$$

where $\tilde{e}_i \equiv (l_i - a_i)\lambda_{i,min} - d_i$.

Another interpretation for the linear utility would be from a worst-case perspective. The constant a_i can be chosen so as to provide an upper bound for marginal utility, or the slope of the logarithmic function at any given point λ:

$$a_i = \max_{x_i} \frac{\partial U_i^e}{\partial x_i} \Longleftrightarrow a_i = \max_{x_i} \frac{1}{1+x_i} \Rightarrow a_i = 1 \quad , \forall i \tag{2.11}$$

The upper bound value is basing again on the assumption that $x_i \geq 0$ for all i. The parameter d_i is the same as in Equation (2.5), and since it is a constant, it can be ignored in the subsequent optimization step. It will be shown later that, given the flow rates of all other users, x_{-i}, the optimal flow rate of the i^{th} user under linear utility with $a_i = 1$ is always higher than the one under logarithmic utility.

Notice that the same cost function structure (2.10) is arrived at in both local and worst-case analyses, with a_i chosen as described above. Combining the worst-case and local analyses in a single step simplifies the problem at hand significantly.

2.2 Existence and Uniqueness of Nash Equilibrium

We first show the existence of a unique Nash equilibrium for the logarithmic, nonlinear model, described by the cost function (2.6). Next, we give a similar result under the linear utility cost function (2.10). Additionally, we derive the reaction functions for the linear case and calculate the equilibrium point explicitly. We conclude the section with a proposition justifying the use of the worst-case linear utility function analysis.

2.2.1 Uniqueness under logarithmic utility function

The optimization problem of a single i^{th} user, defined as the minimization of the cost function (2.6), is solved under the following constraints:

$$x_i > 0 \tag{2.12}$$

$$x_i < m - x_{-i} \quad ,\forall i \tag{2.13}$$

The first constraint is dictated by the fact that the i^{th} user has requested a flow rate of at least $\lambda_{i,min}$. The second constraint is a physical capacity constraint, which implies that the aggregate sum of all flows at a node cannot exceed its total capacity.

Differentiating the cost function (2.6) of the i^{th} user with respect to x_i everywhere except at $x_i = 0$, we obtain:

$$\frac{\partial J_i(x)}{\partial x_i} = \begin{cases} \frac{kx_i^2 + 2k_i x_i[m - (x_i + x_{-i})]}{[m - (x_i + x_{-i})]^2} - \frac{1}{1 + x_i}, & x_i > 0 \\ \frac{kx_i^2 + 2k_i x_i[m - (x_i + x_{-i})]}{[m - (x_i + x_{-i})]^2} - \frac{\partial f(x_i + \lambda_{i,min})}{\partial x_i}, & x_i < 0 \end{cases} \quad ,\forall i \tag{2.14}$$

Notice that $\frac{\partial f(\lambda_i)}{\partial x_i}$ is nonnegative by the definition of $f(\lambda_i)$. Hence, (2.14) attains negative values in the interval $x_i < 0$, for each i. Moreover, the cost at $x_i = 0$ is always higher than the cost in the region $x_i > 0$, since the function $f(\lambda_i)$ is bounded above by the constant $d_i > 0$. In other words, a user can decrease the cost by increasing the flow. By the definition of Nash equilibrium, a single user cannot improve his situation at equilibrium by unilaterally changing his own strategy, or flow rate in this case, given the flow rates of other users. Thus, the optimal point x_i^* has to be strictly positive.

For the second constraint, it can be observed that the cost function (2.6) of the i^{th} user becomes positive unbounded as x_i^* approaches $m - x^*_{-i}$. Again, the user can decrease its cost unilaterally by decreasing his flow rate, and hence the boundary point $x_i^* = m - x^*_{-i}$ cannot be an optimal point. The conclusion, therefore, is that every Nash equilibrium has to be an inner solution.

Theorem 2.1 *There exists a unique Nash equilibrium in the network game with users having a logarithmic utility function.*

Proof: Let r be an M-dimensional vector with all components positive. Define the pseudogradient vector:

$$g(x,r) = \begin{pmatrix} r_1 \nabla_{x_1} J_1(x_1, x_{-1}) \\ \vdots \\ r_M \nabla_{x_M} J_M(x_M, x_{-M}) \end{pmatrix} \tag{2.15}$$

In view of Theorem 2 of Rosen [13], there exists a unique Nash equilibrium, if

$$\sum_{i=1}^{M} (x_i^1 - x_i^0) r_i [\nabla_{x_i} J_1(x_i^0, x_{-i}^0) - \nabla_{x_i} J_1(x_i^1, x_{-i}^1)] > 0 \tag{2.16}$$

for any two flow vectors x^1 and x^0 with elements x_i^0 and x_i^1 constrained according to (2.12) and (2.13).

Define $G(x,r)$ as the Jacobian of $g(x,r)$ with respect to x. In order for condition (2.16) to hold, it is sufficient to show that the symmetric matrix $[G(x,r) + G'(x,r)]$ is positive definite (see Theorem 6 in Rosen [13]). Hence, positive definiteness of $[G(x,r)+G'(x,r)]$ is a sufficient condition for the existence and uniqueness of a Nash equilibrium.

Differentiate the cost function (2.6) for the i^{th} user twice with respect to x_i and define B_i as

$$\begin{aligned} B_i &\equiv \frac{\partial^2 J(x)}{\partial x_i^2} \\ B_i &\equiv \frac{4k_i x_i}{[m-(x_i+x_{-i})]^2} + \frac{2k_i x_i^2}{[m-(x_i+x_{-i})]^3} + \frac{2k_i}{[m-(x_i+x_{-i})]} + \frac{1}{(1+x_i)^2} \end{aligned} \tag{2.17}$$

where $x_i \geq 0$. Also define $A_{i,j}$ in the same region for any $1 \leq i,j \leq M$ with $j \neq i$, as:

$$A_{i,j} \equiv \frac{\partial^2 J_i(x)}{\partial x_i \partial x_j} = \frac{2k_i x_i}{[m-(x_i+x_{-i})]^2} + \frac{2k_i x_i^2}{[m-(x_i+x_{-i})]^3} \tag{2.18}$$

To simplify the notation, we drop the indices i and j for the rest of analysis as it applies to all users. The pricing parameter was defined as $k_i > 0$. Using the constraints (2.12) and (2.13), we obtain the following:

$$A > 0 \quad , \; B > 0 \quad , \; B > A \tag{2.19}$$

It is now sufficient to show the positive definiteness of $[G(x,r) + G'(x,r)]$ for some vector r. A natural choice for r is $r = [1\,1 \ldots 1]'$. From (2.17) and (2.18) we obtain:

$$[G(x,1) + G'(x,1)] = 2 \begin{pmatrix} B & A & \cdots & A \\ A & B & & A \\ \vdots & & \ddots & \vdots \\ A & A & \cdots & B \end{pmatrix}_{M \times M} =: 2(\mathcal{G}_1 + \mathcal{G}_2) \qquad (2.20)$$

where the $M \times M$ matrices $\mathcal{G}_1$ and $\mathcal{G}_2$ are defined as:

$$\mathcal{G}_1 := (B - A)\mathcal{I}\ , \quad \mathcal{G}_2 := A \begin{pmatrix} 1 & \cdots & 1 \\ \vdots & \ddots & \vdots \\ 1 & \cdots & 1 \end{pmatrix}_{M \times M} \qquad (2.21)$$

where $\mathcal{I}$ is the $M \times M$ identity matrix. Since $B - A > 0$, the matrix $\mathcal{G}_1$ is positive definite. Furthermore, since $A > 0$, $\mathcal{G}_2$ is nonnegative definite, with one positive and $M - 1$ zero eigenvalues. Hence (2.20) is positive definite, and the sufficient condition of Rosen [13] for existence of a unique Nash equilibrium is satisfied.

Finally, we check the boundary conditions. First set of boundary points are the ones where $x_i = 0$ for one or more users. Since these users can decrease their costs by increasing their flow rate as indicated in (2.14), this set of points fail as equilibrium points by definition. Similarly, the second set of boundary points, for which $\sum x_i^* = m$, do not qualify as a Nash equilibrium. In conclusion, there exists a unique, feasible Nash equilibrium which is the inner solution for the given constraints.

2.2.2 Uniqueness under linear utility function

Here, we show the existence of a unique Nash equilibrium for the cost function with linear utility. Furthermore, exploiting the linearity of reaction functions, we calculate the equilibrium point explicitly. The analysis in this section applies not only to the worst-case analysis but also to the local analysis, where the logarithmic utility function is approximated by a linear function.

Again, each user minimizes his cost function (2.6), subject to the constraints given in (2.12) and (2.13). First, assuming an inner solution, we have for the i^{th} user

$$\frac{\partial J_i(x)}{\partial x_i} = \frac{kx_i^2 + 2k_i x_i m - 2k_i x_i (x_i + x_{-i})}{(m - (x_i + x_{-i}))^2} - a_i = 0 \tag{2.22}$$

which can be solved for x_i, to lead to

$$x_i = (m - x_{-i})[1 \pm \sqrt{\frac{k_i}{k_i + a_i}}] \tag{2.23}$$

The solution with the plus sign is eliminated in view of the constraint $m - x_{-i} \geq x_i$; hence, the only feasible solution is the one with the minus sign

$$x_i = (m - x_{-i})[1 - \sqrt{\frac{k_i}{k_i + a_i}}] \equiv mq_i - q_i x_{-i} \tag{2.24}$$

where

$$q_i \equiv 1 - \sqrt{\frac{k_i}{k_i + a_i}} \tag{2.25}$$

To complete the derivation, we now check the boundary solutions. For the boundary point $x_i = 0$, we observe from (2.22) that $\frac{\partial J^i(x)}{\partial x_i} = -a_i$, which means the user can decrease his cost by increasing x_i. Hence, this cannot be an optimal point. For the other boundary point $x_i = m - x_{-i}$, we observe that at that point the cost goes to infinity. As a result, the inner solution is the unique optimal point for the constrained optimization problem of the i^{th} user, for each fixed $x_{-i} < m$. We observe from (2.24) that the unique optimal flow for the i^{th} user is a linear function of the aggregate flow of all other users. This set of M equations can now be solved for x_i, $i = 1, \ldots, M$. To ease the notation, let $\bar{x} := x_i + x_{-i}$. Then, (2.24) can be rewritten as

$$x_i = mq_i - q_i(\bar{x} - x_i) \Rightarrow x_i = \frac{q_i}{1 - q_i} m - \frac{q_i}{1 - q_i} \bar{x}. \tag{2.26}$$

Sum both sides from 1 to M, and let $\lambda := \sum_{i=1}^{M} \frac{q_i}{1-q_i}$. Then,

$$\bar{x} = \lambda m - \lambda \bar{x} \Rightarrow \bar{x} = \frac{\lambda}{1 + \lambda} m \tag{2.27}$$

Note that λ is well defined and positive, since $0 < q_i < 1$,$\forall i$. Hence $\bar{x} < m$, thus satisfying the underlying constraint. Finally, substituting $\bar{x}$ above into the expression for x_i (in terms of $\bar{x}$), yields the following unique solution to (2.24):

$$x_i^* = \frac{1}{1+\lambda}\frac{q_i}{1-q_i}m \ , \ i = 1, \ldots, M \tag{2.28}$$

Note that (2.28) is feasible since it is strictly positive, and $\sum_{i=1}^{M} x_i^* < m$. We summarize this result in the following theorem, whose proof follows from the foregoing derivation:

Theorem 2.2 *There exists a unique Nash equilibrium in the network game with users having linear utility functions, and it is given by (2.28).*

We conclude the section with an important proposition, justifying the worst-case analysis based on linear utility functions.

Proposition 2.1 *Given the total flow rates of all users except the i^{th} one, $x_{-i} = \sum_{j \neq i} x_j$, the optimal flow rate of the i^{th} user, $x_{i,nonlin}^{opt}$, having a logarithmic utility and the cost function (2.6) is less than the rate $x_{i,lin}^{opt}$ when the same user has the linear utility (2.8) with $a_i = 1$ and cost function (2.10).*

Proof: The optimal solution of the i^{th} user was already shown to be an inner solution. Differentiating the linear utility cost, $J_{i,lin}$, given by (2.10), and the logarithmic utility cost, $J_{i,nonlin}$, given by (2.6), both with respect to x_i, we obtain

$$J_{i,lin}^{'} = P_i^{'} - U_{i,lin}^{'} = P_i^{'} - 1 \tag{2.29}$$

$$J_{i,nonlin}^{'} = P_i^{'} - U_{i,nonlin}^{'} = P_i^{'} - a_i, \ 0 < a_i \leq 1 \tag{2.30}$$

where a prime denotes partial derivative with respect to x_i. The pricing function P_i in (2.3) is unimodal with a global minimum at $x_i = 0$. Hence, for $x_i \geq 0$, $P_i^{'}$ is a monotone increasing function passing through the origin. Hence, the point at which (2.29) is zero is no smaller than the point at which (2.30) is zero, that is, $x_{i,nonlin}^{opt} \leq x_{i,lin}^{opt}$. This completes the proof.

The intuitive explanation of this result lies in the high marginal demand of worst-case utility, $a = 1$. The marginal demand of a user with linear utility is higher than the one with logarithmic utility. The proposition above is based on this difference in demand.

2.3 Update Algorithms and Stability

In the previous section, it was shown that a unique equilibrium point exists under different cost functions, where each user attains a minimum cost, given flow rates of the other users. In a distributed environment, however, each user acts independently and convergence to this point does not occur instantenously. There exist various iterative update schemes with different convergence and stability properties [6]. We consider here three asynchronous update schemes relevant to the proposed model: PUA (parallel update algorithm), which is also known as Jacobi algorithm; RUA (random update algorithm); and GUA (gradient update algorithm), also known as Jacobi overrelaxation [14]. For the specific model at hand, individual users do not need to know the specific flow rates of other users, except their sum. This feature is of great importance for possible applications as it simplifies the information flow within the system substantially.

2.3.1 Parallel update algorithm

In PUA, the users optimize their flow rates at each iteration in discrete time intervals $\ldots n-1, n, n+1 \ldots$. If the time intervals are chosen to be longer than twice the maximum delay in the transmission of flow information, it is possible to model the system as an ideal, delay-free one. In a system with delays, there are subsets of users, updating their flows given the delayed information on the flow.

One important feature of PUA is that the users are myopic. They optimize their flow rates based on instant costs and parameters, ignoring future implications of their actions. In a delay-free system, this behavior affects convergence rate adversely as it will be seen in the simulations.

For the case of the nonlinear cost function (2.6), the players use either nonlinear programming methods to minimize their cost at each iteration, or they use the reaction function directly. The analytical solution to the optimization problem of the i^{th} user turns out to be the root of the third-order equation:

$$k_i x_i^3 + (2k_i(m - x_{-i}) + k - 1)x_i^2 + (2(k+1)(m - x_{-i}))x_i + [m - x_i]^2 = 0 \qquad (2.31)$$

Only one root of this equation, denoted $\tilde{x}_i$, is feasible: $0 < \tilde{x}_i < m$. The closed-form solution for this root is at the same time the reaction function, which is highly nonlinear in contrast to the linear reaction function given by (2.24). As the root of (2.31) involves a complicated expression, we write the nonlinear reaction function of the i^{th} user only symbolically:

$$x_i^{(n+1)} = f(x_{-i}^{(n)}, k_i) \qquad (2.32)$$

Stability and convergence of the system is as important as the existence of a unique equilibrium. In an unstable system, the flow rates may oscillate indefinitely if there is a deviation from equilibrium. Or, if the system does not have the global convergence property, there exists the possibility of not reaching the equilibrium at all through iteration starting at an arbitrary feasible point. We now study the convergence of PUA. For the linear case, the update function for the i^{th} user is (from (2.24))

$$x_i^{(n+1)} = mq_i - q_i x_{-i}^{(n)} \quad \forall i, n \qquad (2.33)$$

where q_i was defined in (2.25). Let $\triangle x_i = x_i - x_i^*$, where x_i^* is the flow rate of the i^{th} user at Nash equilibrium and $\triangle x_i^{(n)}$ is the difference between the user's flow rate at the n^{th} instant and its final equilibrium flow. Then we have

$$\triangle x_i^{(n+1)} = -q_i \triangle x_{-i}^{(n)} \quad , \forall i \qquad (2.34)$$

Let

$$\|\Delta x\| = \max_i |\Delta x_i| \qquad (2.35)$$

and note that, from (2.34):

$$\|\Delta x^{(n+1)}\| \leq (M-1)\max_i |q_i|\, \|\Delta x^{(n)}\| \tag{2.36}$$

Clearly, we have a contraction mapping in (2.34) if $(M-1)\max_i |q_i| < 1$. Thus, the following sufficient condition ensures the stability of the system with linear utility under the PUA algorithm:

$$|q_i| \leq \frac{1}{M}\ ,\ i = 1, \ldots, M \tag{2.37}$$

One trivial way of meeting this condition is to set $q_i = \frac{1}{M}\ \ , i = 1, \ldots, M$. From (2.25), (2.37) translates into the following stability constraint on the pricing parameters, k_i :

$$k_i \geq \frac{(M-1)^2}{2M-1} a_i \tag{2.38}$$

Notice that these apply not only to the analysis in the linear-utility case but also to the local analysis of the nonlinear-utility cost function (2.6). Thus, the system is locally stable and convergent under PUA if the condition above is satisfied. Next, we show that the system not only has local stability and convergence property but also is globally stable and convergent for the nonlinear cost. Before proceeding with the proof, we present the following two useful lemmas.

Lemma 2.1 *For any feasible point* $\mathbf{x_o} = \mathbf{x^{(n)}}$ *at time* n*, let* $x_i^{(n+1)}$ *be the outcome of the* i^{th} *user's nonlinear reaction function (2.32). If* $x_i^{(n)} > x_i^*$*, where* $\mathbf{x}^*$ *is the unique equilibrium, then* $x_i^{(n+1)} \leq x_i^{(n)}$*.*

Proof: Assume that $x_i^{(n+1)} > x_i^{(n)} > x_i^*$. Given the flow rate, $x_i^{(n)}$, of the i^{th} user at any time instant n, we linearize the logarithmic utilility U_i given in (2.5) around this value, and turn it into (2.8) by defining

$$a_i := \frac{\partial U_i^{(n)}}{\partial x_i} = \frac{1}{1 + x_i^{(n)}} \tag{2.39}$$

Thus, the nonlinear reaction function (2.32) is linearized to (2.33) at $x_i^{(n)}$. Using $a_i > \frac{\partial U_i^{(n+1)}}{\partial x_i}$ and following an argument similar to that in Proposition 2.1, the resulting flow $x_{i,lin}^{(n+1)}$ provides an upper bound on $x_i^{(n+1)}$. Combined with the contraction property of linear reaction function, we obtain

$$x_i^{(n+1)} < x_{i,lin}^{(n+1)} < x_i^{(n)} \tag{2.40}$$

Obviously, (2.40) contradicts the assumption made, and hence $x_i^{(n+1)} \leq x_i^{(n)}$.

Lemma 2.2 *For any feasible point* $\mathbf{x_o} = \mathbf{x^{(n)}}$ *at time* n*, if* $x_i^{(n)} < x_i^*$*, then* $x_i^{(n+1)} \geq x_i^{(n)}$.

Proof: The proof is very similar to the one of Lemma 2.1. Suppose that $x_i^{(n+1)} < x_i^{(n)} < x_i^*$. Then, it can be shown that $a_i < \frac{\partial U_i^{(n+1)}}{\partial x_i}$, and $x_{i,lin}^{(n+1)}$ provides a lower bound on $x_i^{(n+1)}$. Again using the contaction property,

$$x_i^{(n+1)} > x_{i,lin}^{(n+1)} > x_i^{(n)} \tag{2.41}$$

As (2.41) contradicts the initial hypothesis, $x_i^{(n+1)} \geq x_i^{(n)}$.

Theorem 2.3 *The system is globally convergent and stable under PUA, for both the linear and logarithmic utility cost functions (2.6) and (2.10), under the sufficient condition (2.38).*

Proof: The convergence result for the linear utility case was obtained above. In principle, it is also possible to derive the global convergence result for the logarithmic utility case, using the same method, and the reaction function (2.32) is used instead of the reaction function of the linear case. The reaction function (2.32) is obtained as one of the roots of the third order Equation (2.31) and is highly nonlinear. Hence the method based on reaction functions becomes practically intractable. We will, therefore, make use of the local and worst-case analysis to obtain the global convergence result. As in Lemma 2.1, the nonlinear reaction function (2.32) can be linearized to (2.33) at $x_i^{(n)}$. Also, note that the existence of a unique feasible Nash equilibrium, $0 < x_i^* < m$, was already established for the network game.

For any feasible initial point $\mathbf{x_0}$ we have the following cases for the i^{th} user:

In the first case, $x_i^{(n)} > x_i^*$, where $x_i^{(n)} = x_{i,0}$ is the starting point. According to Lemma 2.1, there are two possibilities: $x_i^* < x_i^{(n+1)} < x_i^{(n)}$ and $x_i^{(n+1)} < x_i^* < x_i^{(n)}$. The former case results in a monotone decreasing sequence bounded below by x_i^*, which therefore converges to the equilibrium. The latter case leads to an oscillating sequence around the equilibrium.

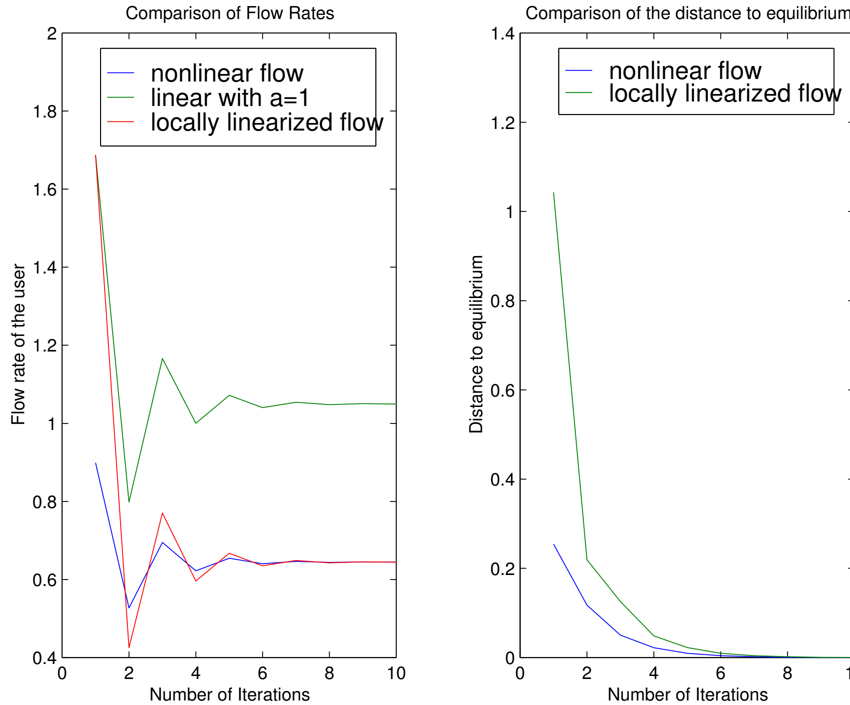
In the second case, $x_i^{(n+1)} < x_i^*$, where $x_i^{(n+1)}$ can be considered as the starting point at any time instant $(n+1)$. Again by Lemma 2.2, there are two possibilities: $x_i^* > x_i^{(n+1)} > x_i^{(n)}$ and $x_i^{(n+1)} > x_i^* > x_i^{(n)}$. Similar to the previous case, the former leads to a monotone increasing sequence bounded above by x_i^*, thus converging to the equilibrium. The latter results again in an oscillating sequence around the equilibrium. In order to analyze it, one can define the relative distance to equilibrium as $\triangle x_i^{(n)} := x_i^{(n)} - x_i^*$.

If $x_i^{(n+1)} > x_i^{(n)}$, then the linearized reaction function at $x_i^{(n)}$ provides an upper bound, $x_{i,linear}^{(n+1)}$, on $x_i^{(n+1)}$, following an argument similar to the one in Proposition 2.1. The fact that $\frac{\partial U_i^{(n)}}{\partial x_i} < a_i^{(n)}$ justifies the given bound. Using the contraction property of linearized reaction function (2.34) and the worst-case bound above, we obtain

$$\triangle x_{i,linear}^{(n+1)} < \triangle x_i^{(n)} < \triangle x_{i,linear}^{(n)} \quad , \forall i \tag{2.42}$$

For $x_i^{(n+1)} < x_i^{(n)}$, following a similar argument, it is easy to show that the relation (2.42) also holds. Therefore, in the case of an oscillating sequence around the equilibrium, we have shown that, at each iteration locally linearized flows provide a decreasing upper bound to the iterates of the nonlinear reaction function for the distance to equilibrium. Figure 2.2 summarizes this discussion graphically.

The flow rate of any i^{th} user converges to the unique Nash equilibrium and the nonlinear system is stable and globally convergent from any feasible initial point $\mathbf{x_0}$. We note that the proof is based on the convergence of the linear system, which is required for the convergence of the nonlinear system. Moreover, condition (2.37), or equivalently (2.38), is sufficient for the convergence of the iteration corresponding to linear and nonlinear reaction functions.



(a) (b)

Figure 2.2 Comparison of nonlinear, locally linearized: $a_i = \frac{1}{1+x_i}$ and linear worst-case: $a_i = 1$ flow rates (a). Distance to equilibrium, nonlinear and locally linearized (b).

2.3.2 Random update algorithm

RUA is a stochastic modification of PUA. The users optimize their flow rates in discrete time intervals and infinitely often, with a predefined probability $0 < p_i < 1$. Thus, at each iteration a random set of users among the M update their flow rates. Again, the users are myopic and make instantaneous optimizations. In the limiting case, $p_i = 1$, RUA is the same as PUA. The nonideal system with delay is also similar to PUA. The users make decisions based on delayed information at the updates, if the round trip delay is longer than the discrete time interval.

For the linear-utility case (2.6) with linear reaction function (2.24), the update scheme may be formulated for the i^{th} user as follows:

$$x_i^{(n+1)} = \begin{cases} mq_i - x_{-i}^{(n)} q_i & , with\ probability\ p_i \\ x_i^{(n)} & , with\ probability\ 1 - p_i \end{cases} \tag{2.43}$$

Subtracting x_i^* from both sides, we obtain

$$\triangle x_i^{(n+1)} = \begin{cases} -q_i \triangle x_{-i}^{(n)} & , with\ probability\ p_i \\ \triangle x_i^{(n)} & , with\ probability\ 1 - p_i \end{cases} \tag{2.44}$$

Taking the absolute value of both sides, and then taking expectations, lead to

$$\begin{aligned} E|\triangle x_i^{(n+1)}| \leq & \quad p_i q_i E|\triangle x_{-i}^{(n)}| + (1 - p_i) E|\triangle x_i^{(n)}| \\ \leq & \quad p_i q_i \textstyle\sum_{j=1}^{M} E|\triangle x_i^{(n)}| + (1 - p_i(1 + q_i)) E|\triangle x_i^{(n)}| \end{aligned} \tag{2.45}$$

Choosing $p_i \geq \frac{1}{1+q_i}$, this can further be bounded by

$$E|\triangle x_i^{(n+1)}| \leq p_i \cdot q_i \sum_{j=1}^{M} E|\triangle x_j^{(n)}| \tag{2.46}$$

and summing over all users we obtain

$$\sum_{i=1}^{M} E|\triangle x_i^{(n+1)}| \leq (\sum_{i=1}^{M} p_i \cdot q_i) \cdot \sum_{i=1}^{M} E|\triangle x_i^{(n)}| \tag{2.47}$$

If $\sum_{i=1}^{M} p_i q_i < 1$, $\mu^{(n)} := \sum_{i=1}^{M} E|\triangle x_i^{(n)}|$ is a decreasing positive sequence, and hence converges to zero. This implies convergence of each individual term in the summation to zero, which in turn says that $x_i^{(n)} \rightarrow x_i^*$, $i = 1, \ldots, M$, with probability 1. Notice that the sufficient condition for the stability of PUA (2.37) also guarantees the stability of RUA for the linear utility case.

The question now comes up as to the choice of p_i that would lead to fastest convergence in (2.47), which we will call the optimal update probability. Maheswaran and Başar [15] shows that in a quadratic system without delay, one can find a bound for optimal update probability $p_{opt} \leq \frac{2}{3}$, as number of users goes to infinity. Repeating the same analysis for this model and linear cost function leads to an exact update probability $p_{opt} = \frac{2}{3}$, which is optimal for a large number of users.

The stability and convergence results obtained also apply to the local analysis of the nonlinear utility function as in PUA. Hence the nonlinear utility case is locally stable under RUA. Moreover, Lemma 2.1 and Lemma 2.2 are valid for RUA, and hence Theorem 2.3 holds, indicating the global stability of the algorithm.

2.3.3 Gradient update algorithm

GUA can be described as a relaxation of PUA. For this scheme, we define a relaxation parameter s_i , $0 < s_i < 1$ for the i^{th} user, which determines the step size the user takes towards the equilibrium solution at each iteration.

For the linear utility case, the algorithm is defined as:

$$x_i^{(n+1)} = x_i^{(n)} + s_i \cdot [(mq_i - x_{-i}^{(n)} q_i) - x_i^{(n)}] \;\; \forall i, n \tag{2.48}$$

Different from both PUA and RUA, the users are not myopic in this scheme. Although they seem to choose suboptimal flow rates at each iteration instead of exact optimal solutions, they benefit from this strategy by reaching equilibrium faster. GUA, despite its deterministic nature like PUA, is very similar to RUA in analysis. When compared with PUA, as we observe in simulations, GUA converges faster to Nash equilibrium than PUA in highly loaded delay-free systems, where there is a high demand for scarce resources and users act simultanously. An intuitive explanation can be made using the fact that equilibrium point is quite dynamic in loaded systems during iterations. In PUA, users update their flows as if it is static, while in GUA, users behave more cautiously and do not rush to the temporary equilibrium point at each iteration. So, in GUA we do not see wide fluctuations in flow rates, which, however, can be seen in PUA. One can interpret the relaxation parameter s_i also as a measure of this caution. Another advantage of GUA, its relative insensitivity to delays in the system, can also be explained with the same reasoning.

A similar but deterministic version of the convergence analysis of RUA for the linear utility function yields the same convergence result as in RUA, except that p_i is replaced with s_i:

$$\triangle x_i^{(n+1)} = (1 - s_i)\triangle x_i^{(n)} - s_i \cdot q_i \sum_{j \neq i}^{M} \triangle x_j^{(n)} \tag{2.49}$$

$$\Rightarrow |\triangle x_i^{(n+1)}| \leq (1 - s_i)|\triangle x_i^{(n)}| + s_i \cdot q_i \sum_{j \neq i}^{M} |\triangle x_j^{(n)}| \;, \forall i \tag{2.50}$$

Choosing $s_i \geq \frac{1}{1+q_i}$, and imposing the condition $\sum_{i=1}^{M} s_i q_i < 1$, the flow rates of the users converge to the unique equilibrium as in other schemes. Using (2.48), we obtain:

$$\text{As n} \to \infty \quad \mathrm{x_i^{(n+1)} = x_i^{(n)} \Rightarrow x_i^* = mq_i - x_{-i}^* q_i} \quad ,\forall \mathrm{i} \tag{2.51}$$

The sufficient condition (2.37) also guarantees the stability of GUA for the linear utility case. Moreover, since the GUA is a modification of PUA, it can be shown that Lemmas 2.1 and Lemma 2.2 hold for the GUA as well. Thus, the stability results of the RUA are directly applicable to GUA for both the linear and nonlinear reaction functions.

Next, we investigate the possibility of finding an optimal relaxation parameter s for the linear utility case, in the sense that it leads to fastest convergence to the equilibrium. In order to simplify the analysis we assume symmetric users, resulting in $q_i = q = \frac{1}{M}$, and $s_i = s$,$\forall i$. For the special case of symmetric initial conditions, we obtain from (2.48):

$$\triangle x_i^{(n+1)} = [1 - s(1 + (M-1)q)] \triangle x_i^{(n)} \tag{2.52}$$

The value of s, leading to fastest convergence in this case is

$$s_{opt} = \frac{1}{1 + (M-1)/M} \Rightarrow \lim_{M \to \infty} s_{opt} = 0.5 \tag{2.53}$$

which leads to one-step convergence.

For the general case, however, it is not possible to find a unique optimal value of s, as different starting points for users that result in different $\triangle x_i$ at each iteration affect the optimal value of s. Using simulations, we conclude that the optimal value of s for a delay-free linear system should be in the range $0.5 < s_{opt} < 1$.

The analysis for the linear utility case applies to the nonlinear utility case locally, giving the same local stability and convergence results. One can show that in addition to the local results, global convergence and stability of PUA also apply to GUA. Therefore, GUA converges globally to the unique equilibrium in the nonlinear utility case. As will be shown in numerical examples, GUA becomes advantageous only under heavy load, and loses its fast convergence property in lightly loaded systems.

CHAPTER 3

SIMULATION OF THE VARIABLE RATE MODEL

Each update scheme analyzed in the previous section is simulated using MATLAB. The proposed model is tested through extensive simulations for both nonlinear and linear reaction functions. The latter can be considered as either worst-case analysis or local approximation to the nonlinear utility cost. The system is simulated first without delay under all three update schemes: PUA, RUA, and GUA. Next, in the second group of simulations, uniformly distributed delays are added to the system for a more realistic analysis. The convergence rate is measured as the number of iterations required to reach the unique Nash equilibrium. As a simplification, we assumed symmetric users in most cases, where cost parameters like a, k, q, and update probability p for RUA, and relaxation parameter s for GUA are not user specific. Starting condition for simulations is the origin, i.e., zero initial flow, unless otherwise stated. The following criterion is used as the stopping criterion, where M is the total number of users.

$$\sum_{i=1}^{M} |x_i^{(n+1)} - x_i^{(n)}| \leq M \cdot \epsilon \tag{3.1}$$

The stopping distance is chosen sufficiently small, $\epsilon = 10^{-5}$, for accuracy in all simulations.

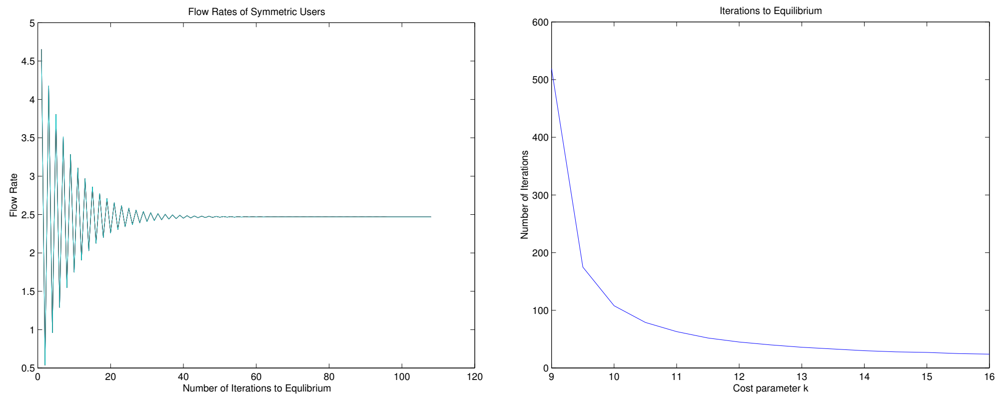


Figure 3.1 Flow rates versus iterations to equilibrium in case of symmetric users and PUA (a). Convergence rate of PUA for different values of k (b).

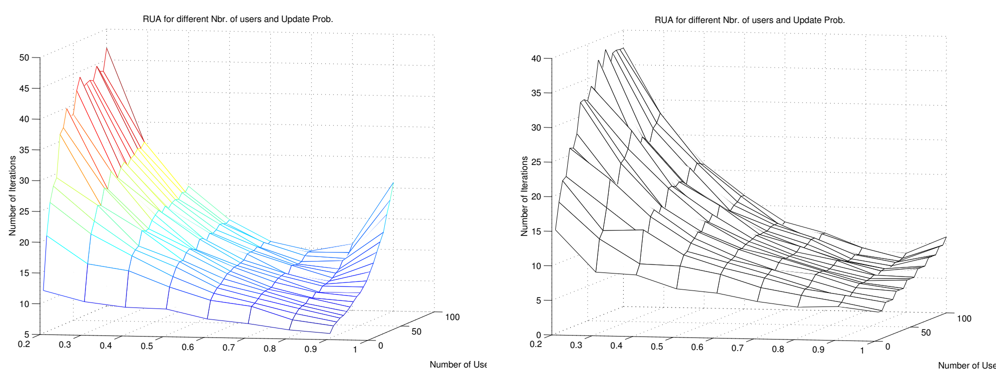
3.1 Simulations for Delay-Free Case

The convergence of the update algorithms for different numbers of users, as a crucial parameter, is investigated throughout the analysis. We first implemented, however, the basic PUA algorithm with $M = 20$ users with linear reaction functions and $a = 1$ indicating a high demand for bandwidth. The price parameter $k = 10$ is chosen to ensure stability. From Figure 3.1(a), we observe the undesirable, wide oscillations in flow rates of users, which is a disadvantage of PUA under a heavily loaded delay-free system. In this case, although the number of users is small, the low value of pricing parameter k loads the system. Absence of delay in the system also contributes to the instantaneous load, as the users act simultaneously. The instantaneous demand affects the convergence rate significantly in delay-free systems, especially under PUA.

Another important parameter in the system is the price, k. The impact of the price on the system is investigated in the next simulation. Figure 3.1(b) shows the effect of varying the pricing parameter k under PUA. Again, there are $M = 20$ users. It can be observed that as the price increases, the convergence rate drops. An intuitive explanation for this phenomenon is based on the effect of price on the demand of users. An increase in price results in a decrease in demand and system load, leading to faster convergence. Even though the simulation here is for a delay-free linear-utility system, varying the price leads to similar results under all update schemes for both linear and nonlinear reaction functions.

Theoretical calculations based on linear utility, in the previous section, show that the minimum value of k satisfying the stability criterion is 9.2 for this specific case. The bound (2.38) is only a sufficient condition for stability, which is verified in this simulation by observing the convergence of system for $k = 9$. The large number of iterations required, on the other hand, indicates the tightness of the bound.

The next set of simulations investigates the two basic parameters of RUA: M, number of active users, and, p, the update probability. The simulation results in Figure 3.2(a) verify the theoretical analysis in the previous section for linear utility cost. It is observed that with the increasing number of users the optimal update probability gets closer to the value $2/3$. For completeness, the same simulation is repeated for the logarithmic utility cost. Interestingly, we obtained similar results, as shown in Figure 3.2(b). Due to the structure of logarithmic utility function, the demand of users is less than in the linear utility case, and hence the system is not loaded as much as in the linear case. For the same number of users, we observe that the optimal update probability shifts to higher values. As a conclusion, Figure 3.2(b) can be considered as a stretched version of Figure 3.2(a), due to the change in load.



(a) (b)

Figure 3.2 Convergence rate of RUA as M gets larger, for different update probabilities $0 < p < 1$, and linear utility (a). Convergence rate of RUA for nonlinear utility (b).

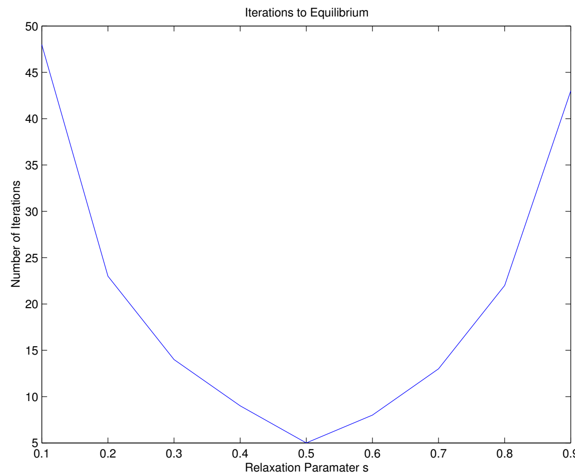


Figure 3.3 Convergence rate of GUA for different values of relaxation parameter, s.

Similar to RUA, a simulation based on the relaxation parameter s is done for linear cost under GUA. The result depicted in Figure 3.3 confirms the theoretical result (2.53) for symmetric initial conditions. Other initial conditions, however, lead to different optimal values for s, in most cases between 0.6 and 0.8. The result can be interpreted as the variation in the amount of instantaneous demand for bandwidth. In the case of symmetric initial condition, all users act the same way, leading to higher simultaneous demand, where "being cautious" or decreasing s is advantageous. For other initial values, the instantaneous demand decreases, where increasing s affects the convergence rate positively. We conclude that GUA is only advantageous in situations with high instantaneous and total demand, which will further be verified in delayed simulations.

Finally, we conclude the simulations without delay by a comparison of the convergence rate of all three algorithms for different numbers of users. The results for the linear reaction function are displayed in Figure 3.4. We observe clearly that both GUA and RUA are superior to PUA. Another important and promising observation is that the rates of convergence for GUA and PUA are almost independent of the number of users. The simulation is repeated for nonlinear utility cost and for highly as well as lightly loaded systems. To change the amount of load on the system, the capacity parameter m is varied this time, instead of price k. They affect, as expected, the convergence rate in opposite ways. Obviously, the smaller the capacity, the heavier the load. Figure 3.5

depicts both situations. Similar to the linear utility case, GUA converges faster with increasing number of users. It performs, however, poorer under light load. Same trend is also observed for RUA. One interesting phenomenon is the high performance of PUA under light load in Figure 3.5. It can be seen as a result of the low instantaneous demand due to the variation of utility for different flow rates.

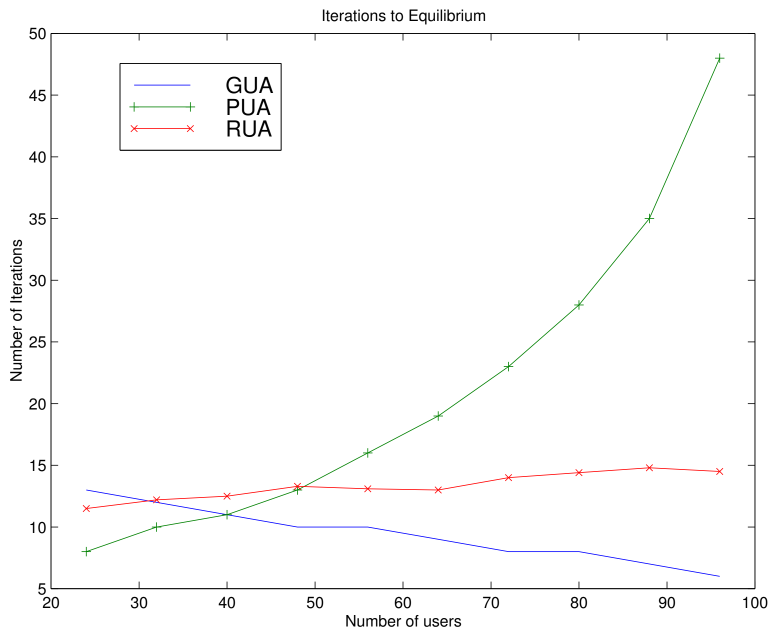


Figure 3.4 Comparison of convergence rates of PUA, RUA, and GUA for increasing number of users, linear utility, delay-free system.

3.2 Simulations with Delay

In order to make the simulations more realistic, we next introduce the delay factor into the system in the following way: users are divided into d equal groups, where each group has an increasing number of units of delay. For example in a four group system, the first group has no delay, the second has one unit delay, the third group two units of delay, etc.

When the simulations are repeated with uniformly distributed delay as described, the results obtained are quite different from the previous ones. PUA, for example, performs

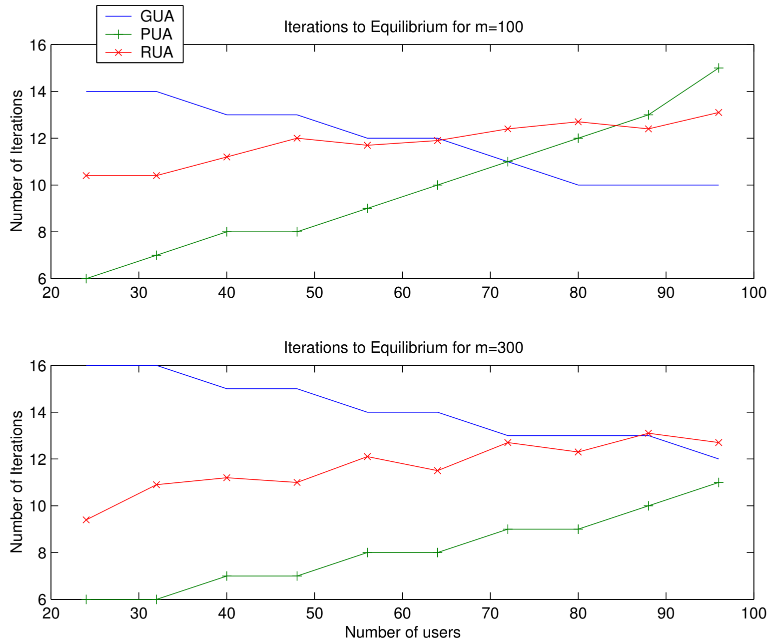


Figure 3.5 Comparison of convergence rates of PUA, RUA and GUA for the nonlinear utility case, in delay-free system.

better than in the linear utility case without delay. This improved result is possibly caused by the decrease of instantaneous demand, due to the delay factor. This result strengthens the argument about PUA in the previous section.

In RUA, however, the optimal update probability disappears in contrast to the delay-free case, as can be seen in Figure 3.6. Again, the underlying cause is the effect of delay factor on instantaneous demand. Another important observation is the similarity of the results for linear (a) and nonlinear (b) cases in this simulation.

Regarding PUA, we can conclude that it performs better when both instantaneous and total demand are low and system resources are abundant. Such conditions exist for delayed systems with users having logarithmic utility. RUA, on the other hand, performs worse with decreasing update probability in such a case.

Next, GUA is investigated under a delay incorporated system for an optimal relaxation parameter. Using the results of several simulations, we conclude that the optimal value of s decreases in the linear utility case, as the delay factor increases. Figure 3.7 shows the effect of s on convergence rate in a delayed system. Interestingly, this trend

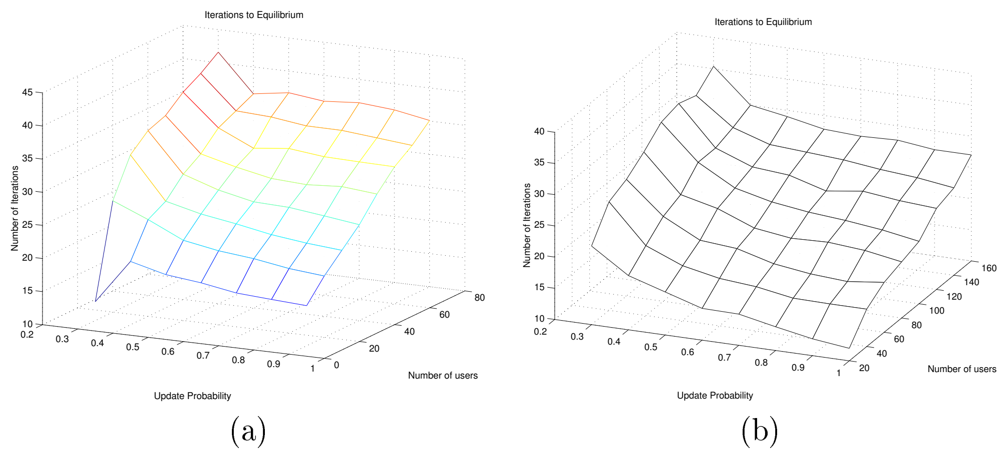


Figure 3.6 Convergence rate of RUA as M gets larger and for different update probabilities $0 < p < 1$ in a delayed system with $d = 5$ (a). Same simulation with the nonlinear system (b).

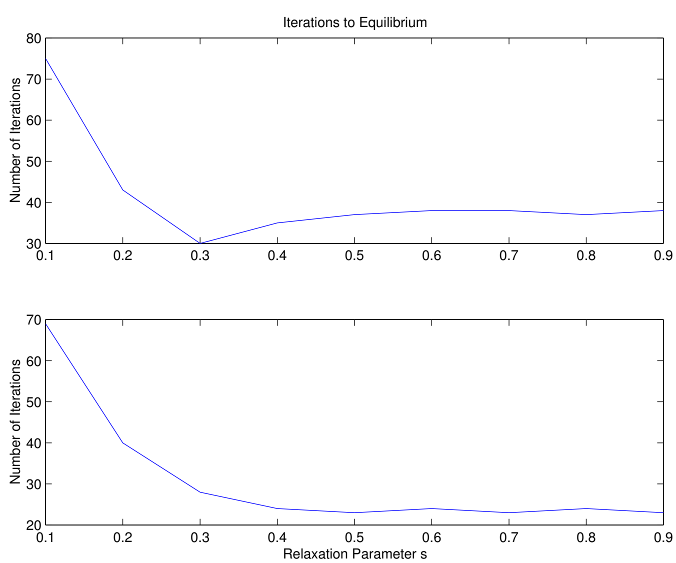


Figure 3.7 Convergence rate of GUA for various values of s in a uniformly delayed system.

disappears for the nonlinear case. Similar to RUA, GUA also loses its advantage with nonlinear reaction functions in a delayed system under normal load.

Comparison of all three algorithms in the delayed nonlinear system for high and low load can be seen in Figure 3.8. In the bottom graph, prices are halved, while the capacity is tripled with respect to the top one. As expected, PUA performs better than RUA for any load. Under light load, PUA is superior to GUA, with the aid of delay factor and low instantaneous delay due to logarithmic utility of users. As the load in the system increases, GUA performs comparable to PUA, verifying the observation in Figure 3.7.

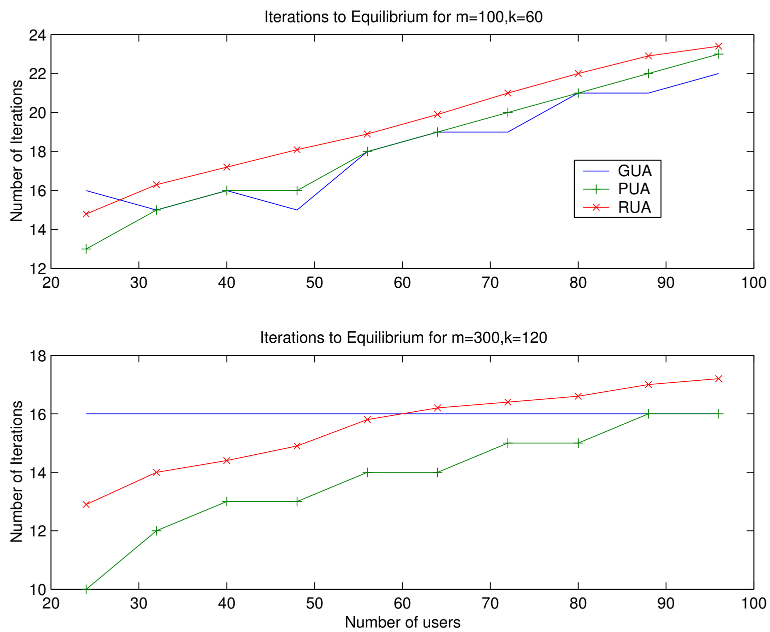


Figure 3.8 Comparison of convergence rates of PUA, RUA and GUA for increasing number of users, nonlinear cost.

In another set of simulations, we investigate the robustness of the algorithms under disturbances. The disturbance is added to the system by varying the number of users at each iteration by about 10% of the total number of users. The arrival and departure of users are modeled as Poisson random processes, and hence, the number of users in the system constitute a Markov chain. Figure 3.9 shows the stability results under different update schemes in terms of the percentage distance to the ideal equilibrium for an example time window. The lower right graph is the result of the simulation with

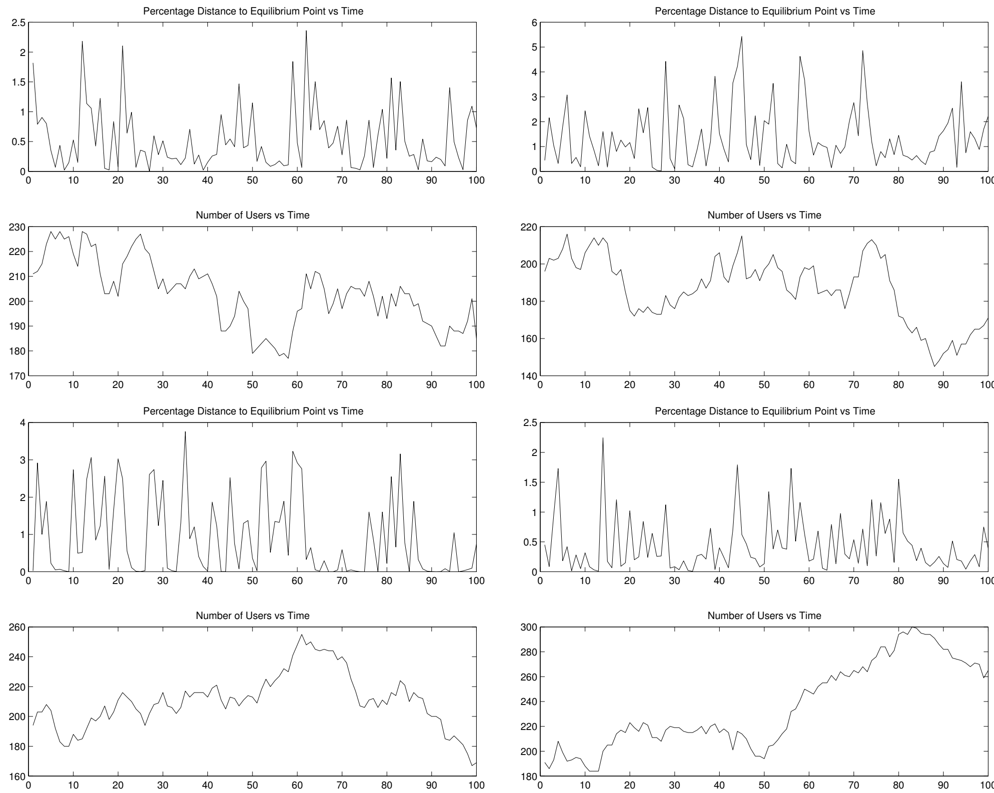


Figure 3.9 Robustness Analysis for PUA, RUA and GUA for linear and PUA for nonlinear utility. Percentage distance and number of users versus time.

nonlinear reaction function under PUA. We observe that the average distances to the equilibrium vary between 0.5% and 1.5%, which indicate that the system is very robust under all schemes and costs.

Finally, a pricing scheme with two classes, a group of priority and a group of regular users is studied. Priority users are charged less in terms of network credits than the regular users by setting the pricing parameter $k = 200$ versus $k = 240$ for the excess flow rate x. A disturbance structure similar to the one above is used by varying the number of users, in order to create a more realistic setting. Again the simulation is done with users having a nonlinear reaction function under PUA. The flow rates of two sample users from each class are shown in Figure 3.10. We observe that the pricing scheme is successful in differentiating the priority user from the regular one. Moreover the robustness of the model is preserved.

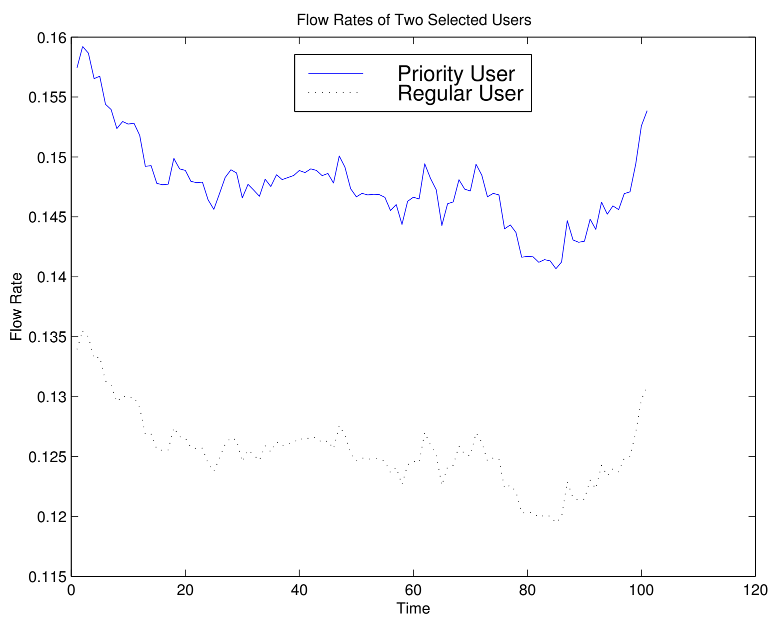


Figure 3.10 Flow rates of a priority user with $k = 200$ and a regular user with $k = 240$ versus time.

CHAPTER 4

A MODEL FOR THE CDMA UPLINK POWER CONTROL

We introduce a game theoretical model for the CDMA uplink power control problem, and establish the existence of a unique Nash equilibrium [1]. Stability of the equilibrium is investigated under two different update schemes: parallel and random update. In addition, a sufficient condition for the stability and convergence of the algorithms is derived. Finally, different pricing schemes, and the relation between price and signal to interference ratio (SIR) of the users are examined.

4.1 The Model and the Cost Function

We propose a simple model for a single cell CDMA system with up to M users. The number of users is limited under an admission control scheme that ensures the minimum necessary SIR for each user in the cell. Defining a power control game, we obtain a distributed and asynchronous control scheme for the uplink power control problem.

For the i^{th} user, we define the cost function J_i by subtracting the utility of the user from its pricing function, $J_i = P_i - U_i$. The utility function is chosen in the standard way as a logarithmic function of the SIR of the user, which we denote by γ_i for user i. The pricing function defines the instantaneous "price" a user pays for using a specific amount of power that causes interference in the system. It is a linear function of $h_i p_i$, the power of the user as received by the base station, where h_i, $0 < h_i \leq 1$, is the user's channel gain. The price, in this context, should be considered in terms of network credits, within a predefined scheme. It is possible, however, to relate the real-world prices to the network credits. The price each user pays is not only adjusted centrally by changing a pricing parameter, λ_i, but it is also proportional to the channel gain of the user, in order to allow distant users with lower channel gains to use higher powers. This pricing scheme aims to achieve fairness among users in the sense that it makes the users insensitive to the the geographical location of the base station. Accordingly, the cost function of the i^{th} user is defined as

$$J_i(p_i, p_{-i}) = \lambda_i \; p_i - \ln(1 + \gamma_i) \quad p_i > 0 \;, \forall i, \tag{4.1}$$

where the SIR function, γ_i, is

$$\gamma_i = L \frac{h_i p_i}{\sum_{i \neq j} h_j p_j + \sigma^2} \tag{4.2}$$

Here, $L = \frac{W}{R}$ is the spreading gain of the CDMA system, where W is the chip rate and R is the total rate. As introduced earlier, the parameter h_j is the channel gain from user j to the base station in the cell.

4.2 Existence and Uniqueness of Nash Equilibrium

The i^{th} user's optimization problem is to minimize its cost, given the sum of powers of other users as received at the base station, $\sum_{i \neq j} h_j p_j$, and the noise. The positivity of the power vector is an inherent physical constraint of the model, $p_i \geq 0 \;\; , \forall i$. Taking the derivative of the cost function (4.1) with respect to p_i, we obtain the first-order necessary condition:

$$\frac{\partial J_i(p)}{\partial p_i} = \lambda_i - \frac{L h_i}{\sum_{j \neq i} h_j p_j + L h_i p_i + \sigma^2} \geq 0 \tag{4.3}$$

Assuming a positive inner solution, (4.3) holds with equality. It is easy to see that the second derivative is also positive, and hence the inner solution, if it exists, is the unique point minimizing the cost function. The boundary solution, $p_i = 0$, is the other possible optimal point for the constrained optimization problem. If the user's cost function, $J_i(p_i, p_{-i})$, attains its minimum for a power value less than zero, $p_{i,min} < 0$, the optimal solution will be the boundary point. Solving Equation (4.3) and applying the positivity constraint $p_i \geq 0$, we obtain the reaction function of i^{th} user:

$$p_i(p_{-i}, \lambda_i) = \begin{cases} \frac{1}{\lambda_i} - \frac{1}{Lh_i}(\sum_{j \neq i} h_j p_j + \sigma^2) & , \quad if \sum_{j \neq i} h_j p_j \leq \frac{Lh_i}{\lambda_i} - \sigma^2 \\ 0 & , \quad else \end{cases} \tag{4.4}$$

We observe two conditions on the price of i^{th} user: λ_i (from (4.4)) in order for the mobile to be "active," or $p_i > 0$. The first condition comes from the fact that both channel gains and powers of users are positive, $h_j, p_j > 0 \;\; , \forall j$. This results in an upper bound on λ_i in terms of system parameters L and σ^2 as well as the channel gain of the user, h_i:

$$\lambda_i < \frac{Lh_i}{\sigma^2} \;\; , \forall i \tag{4.5}$$

The second condition, $\sum_{j \neq i} h_j p_j \leq \frac{Lh_i}{\lambda_i} - \sigma^2$, given in (4.4) also applies based on (4.5). Both of these conditions need to be satisfied in order for the mobile to be active with $p_i > 0$. An intuitive interpretation for these conditions is, If the price λ_i is set too high for a mobile, the mobile prefers not to transmit at all, depending on his utility and cost functions. In reality, positivity of power is only a necessary, and not a sufficient condition for a mobile to establish communication with the base station. Setting a lower bound on SIR is more realistic. The relationship between SIR, number of users, and price will be further investigated for different pricing schemes.

For any equilibrium solution, the set of fixed point equations can be written in matrix form by exploiting the linearity of (4.4). In case of a boundary solution, the rows and columns of users with zero equilibrium power are deleted, and the matrix equation contains only users with positive powers.

$$\begin{pmatrix} 1 & \frac{h_2}{Lh_1} & \frac{h_3}{Lh_1} & \cdots & \frac{h_M}{Lh_1} \\ \frac{h_1}{Lh_2} & 1 & \frac{h_3}{Lh_2} & \cdots & \frac{h_M}{Lh_2} \\ \frac{h_1}{Lh_3} & \frac{h_2}{Lh_3} & 1 & \cdots & \frac{h_M}{Lh_3} \\ \vdots & \vdots & & \ddots & \vdots \\ \frac{h_1}{Lh_M} & \frac{h_2}{Lh_M} & \cdots & \frac{h_{M-1}}{Lh_M} & 1 \end{pmatrix} \begin{pmatrix} p_1^* \\ \vdots \\ p_i^* \\ \vdots \\ p_M^* \end{pmatrix} = \begin{pmatrix} \frac{1}{c_1 h_1} \\ \vdots \\ \frac{1}{c_i h_i} \\ \vdots \\ \frac{1}{c_M h_M} \end{pmatrix} \Leftrightarrow \mathcal{A}\mathbf{p}^* = \frac{1}{\mathbf{ch}} \;, \tag{4.6}$$

where the constant c_i is defined as $c_i = L\lambda_i/(L - \sigma^2\lambda_i)$.

Theorem 4.1 *In the power game just defined, there exists a unique Nash equilibrium (NE). Furthermore:*

i. The NE is strictly positive if, and only if, $\min_i \frac{h_i}{\lambda_i} - \frac{1}{L+M-1}\sum_{i=1}^{M}\frac{1}{\lambda_i} > \frac{L-1}{L+M-1}\frac{\sigma^2}{L}$ *and* $\lambda_i < \frac{Lh_i}{\sigma^2}$.

ii. Otherwise, the NE is a unique boundary solution.

Proof: We first show that the matrix $\mathcal{A}$ in Equation (4.6) is full rank and invertible, and hence the solution to (4.6), $\mathbf{p}^*$, is unique. Then we show that the solution is strictly positive if the given condition is satisfied. Finally, we relax the condition and allow for boundary solutions, and conclude the proof by proving the uniqueness of the boundary solution.

In order for the matrix $\mathcal{A}$ in (4.6) to be full rank and hence invertible, there should not exist a vector $y = (y_1, y_2 \dots y_M)^T \neq 0$ such that $\mathcal{A}y = 0$. This expression can be written as the following set of equations:

$$Lh_i y_i + \sum_{j \neq i} h_j y_j = 0 \quad \forall i \tag{4.7}$$

$$\Rightarrow (L-1)h_i y_i + \sum_{j=1}^{M} h_j y_j = 0 \quad \forall i \tag{4.8}$$

Summing up the set of equations over all users $i = 1 \dots M$

$$(L-1+M)(\sum_{j=1}^{M} h_j y_j) = 0 \tag{4.9}$$

It is clear that the term $L-1+M$ in (4.9) is nonzero, and hence the sum $\sum_{j=1}^{M} h_j y_j$ has to be zero. Since the channel gains are strictly positive, $h_i > 0$, it follows from (4.8) that $y_i = 0 \ \forall i$. Accordingly, the matrix $\mathcal{A}$ is full rank and invertible, which results in a unique solution of the equation (4.6).

i. The condition (4.5) was shown to be necessary for the strict positivity of the i^{th} user's power, $p_i^* > 0$, regardless of other users' power levels. Additionally, by rearranging (4.4) at the equilibrium, we obtain the following:

$$(L-1)h_i p_i = (\frac{L h_i}{\lambda_i} - \sigma^2) - \sum_{j=1}^{M} h_j p_j \quad, \forall i \tag{4.10}$$

Without loss of generality, the set of all users can be reduced to the subset, $\tilde{\mathbf{M}}$, of users with positive equilibrium powers, throughout the summations. With a slight abuse of notation, M represents the number of active users for the rest of the proof.

In order to have equilibrium power vector as an inner solution, we need to show that the right-hand side (RHS) of (4.10) is positive for any user. The summation of the equation over all users gives:

$$(L+M-1)\sum_{i=1}^{M} h_i p_i = L\sum_{i=1}^{M} \frac{h_i}{\lambda_i} - M\sigma^2 \tag{4.11}$$

By rearranging (4.11), we obtain the sum of weighted powers in terms of the system parameters:

$$\sum_{i=1}^{M} h_i p_i = \frac{L\sum_{i=1}^{M}\frac{h_i}{\lambda_i} - M\sigma^2}{L+M-1} \tag{4.12}$$

Substituting the expression for $\sum_{i=1}^{M} h_i p_i$ from (4.12) into (4.10), and requiring $h_i p_i > 0 \ \forall i$ leads to the following condition for positivity of the equilibrium power vector:

$$\min_i \frac{Lh_i}{\lambda_i} - \sigma^2 - \frac{L\sum_{i=1}^{M}\frac{h_i}{\lambda_i} - M\sigma^2}{L+M-1} > 0 \tag{4.13}$$

Rewriting this condition, we have

$$\min_i \frac{h_i}{\lambda_i} > \frac{1}{L+M-1}\sum_{i=1}^{M}\frac{h_i}{\lambda_i} + \frac{L-1}{L+M-1}\frac{\sigma^2}{L} \tag{4.14}$$

Thus, the conditions (4.14) and (4.5) are necessary and sufficient for the existence of a unique feasible inner Nash equilibrium.

An interesting observation is that the power level of the i^{th} mobile, (4.4), can be written independent from the other users' power levels at the equilibrium point. From (4.12), the sum of the other mobile's power levels can be replaced with system parameters and prices:

$$p_i = \begin{cases} \frac{1}{h_i}\{\frac{L}{L-1}[\frac{h_i}{\lambda_i} - \frac{1}{L+M-1}\sum_{j\in\tilde{\mathbf{M}}}\frac{h_j}{\lambda_j}] - \frac{1}{L+M-1}\sigma^2\}, \\ \quad if \sum_{j\neq i} h_j p_j \leq \frac{Lh_i}{\lambda_i} - \sigma^2 \\ 0 \;\; , else \end{cases} \tag{4.15}$$

where $\tilde{M}$ is again the number of active users.

Possible boundary solutions need to be investigated to conclude the uniqueness of the inner Nash equilibrium. Assume there exists an i^{th} mobile with zero equilibrium power, and all other mobiles use positive power. In order for this to be a Nash equilibrium, the condition $\sum_{j\neq i} h_j p_j \leq \frac{Lh_i}{\lambda_i} - \sigma^2$ should fail according to the equilibrium power level (4.15) of the mobile. Summing up the equilibrium powers (4.15) of the remaining $\tilde{M} = M-1$ users results in

$$\frac{L}{L-1}[\sum_{j\in\tilde{\mathbf{M}}}\frac{h_i}{\lambda_i} - \frac{\tilde{M}}{\tilde{M}+L-1}\sum_{j\in\tilde{\mathbf{M}}}\frac{h_i}{\lambda_j}] \leq \frac{Lh_i}{\lambda_i} - \frac{L-1}{L+\tilde{M}-1}\sigma^2 \tag{4.16}$$

Rearranging the terms,

$$\frac{1}{L+\tilde{M}-1}\sum_{j\in\tilde{\mathbf{M}}}\frac{h_i}{\lambda_j} < \frac{h_i}{\lambda_i} - \frac{L-1}{(L+\tilde{M}-1)}\frac{\sigma^2}{L} \tag{4.17}$$

From the necessary condition (4.14), $\sum_{j\neq i} h_j p_j \leq \frac{Lh_i}{\lambda_i} - \sigma^2$, is satisfied. Thus, the power of the mobile must be positive. This contradicts the initial assumption, and hence the boundary solution cannot be a Nash equilibrium. An important observation is that equation (4.17) depends on the number of users with positive power. As a result, all boundary solutions fail similarly for being an equilibrium, including the trivial solution, the origin. Furthermore, the condition is also satisfied for the inner solution. We conclude that the inner Nash equilibrium is unique.

ii. If one of the necessary conditions (4.5) or (4.14) is relaxed, clearly the equilibrium will be a boundary point. Specifically, when a user fails to meet (4.5), then its Nash equilibrium will be the zero power level independent from other users. Such users do not affect others and can be ignored. Let us now assume that the users are labeled in such a way that their price to channel gain ratio are in descending order

$$\frac{h_1}{\lambda_1} \geq \frac{h_2}{\lambda_2} \geq \ldots \geq \frac{h_M}{\lambda_M} \tag{4.18}$$

We argue that, if the N^{th} mobile, $1 \leq N \leq M$, fails condition (4.14), then it is dropped together with mobiles $N+1, \ldots, M$. In this case

$$\frac{h_M}{\lambda_M} \leq \cdots \leq \frac{h_{N+1}}{\lambda_{N+1}} \leq \frac{h_N}{\lambda_N} < \frac{1}{L+N-1}\sum_{i=1}^{N}\frac{h_i}{\lambda_i} + \frac{L-1}{L+N-1}\frac{\sigma^2}{L} \tag{4.19}$$

With each dropped user, the number of active mobiles having positive power decreases. M in equation (4.14) represents only the number of active users, so for the $(N+1)^{st}$ user condition (4.14) is same as the RHS of (4.19). Hence, mobiles $N+1, \ldots, M$ fail to satisfy the necessary condition and have zero equilibrium power. For a single user, (4.14) simplifies to the necessary and sufficient condition (4.5).

Note that, when all the users fail to satisfy the conditions, the solution is obviously trivial. A dropping mechanism can be implemented, where users are dropped one by one beginning from the M^{th} user. For symmetric pricing, this corresponds to the mobile with the lowest channel gain, yielding an intuitively justified result.

After removing the row and column entries of the dropped users, the matrix $\mathcal{A}$ remains full rank and invertible. For the rest of the mobiles, the first part of the proof holds as it only depends on the number of active users. Thus, there exists a unique inner Nash equilibrium in the game with only active users. As the users with zero power attain the Nash equilibrium by definition, the original game also admits a unique boundary equilibrium.

4.3 Pricing Strategies

The positivity of a mobile's equilibrium power depends directly on its channel gain according to (4.5). Therefore, the connection might fail if the price is high and channel gain is low, limiting users with higher prices to a smaller area than others within a cell by slow fading of the wireless system. In most of the cases, such a result is not desirable. We propose proportional pricing as a possible solution to this problem. In this scheme, the price is proportional to the channel gain, $\lambda_i = k_i h_i$, allowing users to transmit without being affected by their locations. We will investigate variants of this pricing scheme through the rest of this chapter.

The inner Nash solution by itself does not guarantee that the users with nonzero powers will meet the minimum SIR requirements to establish a connection to the base station. Achieving the necessary SIR level is obviously crucial to the successful operation of the system. Furthermore, increased variety of communication applications in wireless systems leads to different types of users and SIR requirements in addition to the minimum SIR level.

First, we consider the simple symmetric user case with the same SIR requirements. It is possible to find a pricing strategy by formulating the pricing parameter directly proportional to the channel gain, $\lambda_i = kh_i$, where the pricing factor, k, is user independent. It is possible to define k as a function of the number of users and the SIR level.

Because of symmetry, taking $h_i p_i$ and $\frac{h_i}{\lambda_i}$ to be user independent in (4.10), we arrive at

$$(L + M - 1)h_i p_i + \sigma^2 = L\frac{h_i}{\lambda_i} \tag{4.20}$$

Combining this result and the SIR function in Equation (4.2) and taking the minimum SIR, γ^*, as input, we obtain the following pricing strategy for a single class of users in a cell:

$$\lambda_i = h_i[\frac{L}{\sigma^2} \frac{1}{1 - \frac{\gamma^*(M-1)+\gamma^* L}{\gamma^*(M-1)-L}}] \tag{4.21}$$

This strategy for a single class of users meets (4.5), based on the following:

$$\gamma^* < \frac{L}{M-1} \tag{4.22}$$

Moreover condition (4.14) simplifies to (4.5) in the symmetric case. Thus, both necessary and sufficient conditions are satisfied if (4.22) holds. Then the unique solution is strictly positive and all M users attain the desired SIR level. Otherwise, due to symmetry, all users go below the minimum value. In this case, dropping some of the users from the system in order to decrease the effective M would lead to a viable solution.

For the more general case, it is convenient to split the mobiles in a cell into multiple groups according to their need for bandwidth, or in our context, their requested SIR levels. As a result of the inherent nature of CDMA systems, each user affects others, so the users of different groups in a cell cannot be considered separately. Using the multiple pricing scheme, a solution capturing multiple user groups can be formulated. As an example, we will investigate the two group case with symmetric users within each group. It is straightforward to extend the same structure to higher number of user groups with different SIR requirements.

In the multiple pricing scheme, we define the prices $\lambda_i^1 = k^1 h_i$ and $\lambda_j^2 = k^2 h_j$, for each group. Assume one group is for multimedia applications, hence requiring a high SIR. Decreasing the pricing factor k^1 yields the desired result for such users. Relating the real-world prices inversely proportional to the network credits would be a reasonable choice in this case. Thanks to the interaction between users of different groups, the function (4.21) cannot be used to determine the value of the pricing parameters k^1 and k^2. In order to find the appropriate values, we make use of the symmetry of users within a group and derive a relation similar to (4.21).

The target SIR levels for the two groups with N^1 and N^2 users are γ^1 and γ^2, respectively. Rewriting Equation (4.2),

$$\gamma^1 = \frac{L h_i p_i}{N^2 h_j p_j + (N^1 - 1) h_i p_i + \sigma^2} \tag{4.23}$$

$$\gamma^2 = \frac{L h_j p_j}{(N^2 - 1) h_j p_j + N^1 h_i p_i + \sigma^2}, \tag{4.24}$$

$$i = 1, \ldots, N^1 \; ; \; j = 1, \ldots, N^2 \tag{4.25}$$

and solving for $h_i p_i$ and $h_j p_j$ in the set of equations (4.23), we obtain:

$$h_i p_i = \frac{\gamma^1 \sigma^2 (\gamma^2 + L)}{-\gamma^1 \gamma^2 N^1 N^2 + [\gamma^1 (N^1 - 1) - L][\gamma^2 (N^2 - 1) - L]} \tag{4.26}$$

$$h_j p_j = \frac{\gamma^2 \sigma^2 (\gamma^1 + L)}{-\gamma^1 \gamma^2 N^1 N^2 + [\gamma^1 (N^1 - 1) - L][\gamma^2 (N^2 - 1) - L]} \tag{4.27}$$

Finally, we establish the relation between the pricing factors k^1 and k^2 and the power levels within each group as received by the base station, $h_i p_i$ and $h_j p_j$. Notice that the received power level, hence SIR, is the same for all users within the group.

$$k^1 = \frac{L}{(L + N^1 - 1) h_i p_i + (N^2) h_j p_j + \sigma^2} \tag{4.28}$$

$$k^2 = \frac{L}{(L + N^2 - 1) h_j p_j + (N^1) h_i p_i + \sigma^2} \tag{4.29}$$

Combining (4.26)-(4.28) provides the pricing strategy for the two group case in terms of the desired SIR levels, number of users, system parameter L, and noise level σ^2. The set of linear equations can be easily extended to three or higher classes, providing a general pricing strategy.

The expressions in (4.26) may be negative for some mobiles, if it is not possible to achieve the desired SIR level for the given parameters. Thus the Nash equilibrium will be a unique boundary solution according to the second part of Theorem 4.1. By implementing a reasonable dropping mechanism, users can be dropped beginning with the one having lowest channel gain to price ratio, $\frac{h_i}{\lambda_i}$. Then, the remaining users would meet the condition (4.14). Moreover, the following would hold:

$$\frac{\lambda_i^j}{h_i} = \frac{L}{c^j + \sigma^2} < \frac{L}{\sigma^2} \tag{4.30}$$

where $c^j > 0$. Thus, the necessary condition (4.5) is also met and a unique inner Nash equilibrium exists for the game with active users. The solution for all the mobiles, including the inactive ones, is on the other hand a unique boundary solution in accordance with Theorem 4.1.

4.4 Update Schemes and Stability

In this section, we investigate the stability of the Nash equilibrium in the given model under two relevant asynchronous update schemes: parallel and random update, which were defined in Section 2.3. We establish a sufficient condition which guarantees the convergence to the unique equilibrium point for both algorithms.

4.4.1 Parallel update algorithm

In PUA, users optimize their power levels at each iteration using the reaction function defined in (4.4). Depending on the length of the chosen time intervals, the system is characterized either as ideal (i.e., delay-free) or delayed. The algorithm is given by

$$p_i^{(n+1)}(p_{-i}^{(n)}, \lambda_i) = \begin{cases} \frac{1}{\lambda_i} - \frac{1}{Lh_i}(\sum_{j\neq i} h_j p_j^{(n)} + \sigma^2) & , \quad if \sum_{j\neq i} h_j p_j^{(n)} \leq \frac{Lh_i}{\lambda_i} - \sigma^2 \\ 0 & , \quad else \end{cases} \tag{4.31}$$

From Theorem 4.1, it is known that the Nash equilibrium could be a boundary point; i.e., some of the mobiles could have zero equilibrium power. Based on this result, a user-dropping scheme was described in Section 4.2. The power level p of the mobiles dropped from the system remain zero for the given prices and parameters. Hence, these users are stable. For the active users with strictly positive power levels, the distance of the i^{th} user's power, p_i, to the equilibrium at time n is derived using the reaction function in (4.4):

$$p_i^{(n+1)} = \frac{1}{\lambda_i h_i} - \frac{1}{L h_i}\left(\sum_{i \neq j} h_j p_j^{(n)} + \sigma^2\right) \tag{4.32}$$

$$p_i^{*} = \frac{1}{\lambda_i h_i} - \frac{1}{L h_i}\left(\sum_{i \neq j} h_j p_j^{*} + \sigma^2\right) \tag{4.33}$$

$$\Rightarrow |\triangle p_i^{(n+1)}| \leq \frac{1}{L h_i} \sum_{i \neq j} h_j |\triangle p_j^{(n)}| \quad , \forall i \tag{4.34}$$

Using l_1-norm analysis, it can be shown that this is a contraction mapping provided that

$$\max_i \frac{1}{h_i} \sum_{j=1}^{M} \frac{h_j}{L} < 1 \tag{4.35}$$

Hence, (4.35) is a sufficient condition for stability and global convergence of the system. Due to the initial admission control mechanism and user-dropping scheme, which limit the number M of users in the cell, this condition can easily be satisfied for a given set of parameters L and h. Thus, the stability and convergence of the algorithm follows.

4.4.2 Random update algorithm

In RUA, each mobile updates its power level with a predefined probability $0 < P_i < 1$:

$$p_i^{(n+1)} = \begin{cases} p_i^{(n+1)}, & \textit{with probability } P_i \\ p_i^{(n)}, & \textit{with probability } 1 - P_i \end{cases} \tag{4.36}$$

where $p_i^{(n+1)}$ was defined in (4.31). Similar to the case of PUA, mobiles with zero power remain stable for all n and hence are not of any relevance to the ensuing analysis. For the active users, following a derivation similar to the one in (4.32) and taking the expectation, $E[.]$, of the distances to the equilibrium, we obtain

$$E|\triangle p_i^{(n+1)}| \leq \frac{1}{Lh_i} \sum_{i \neq j} h_j E|\triangle p_j^{(n)}| P_i + (1 - P_i) E|\triangle p_i^{(n)}| \tag{4.37}$$

Using l_1-norm analysis, one can show that the condition (4.35) is again sufficient for the right-hand side of (4.37) to be a contraction mapping. Using Gershgorin's theorem [16] one can obtain useful bounds on the eigenvalues of the matrix that defines the right-hand side of (4.37). The inequality that follows directly from Gershgorin's theorem for an eigenvalue ν_i is

$$|\nu_i - (1 - P_i)| \leq \sum_{j \neq i} \frac{h_j}{Lh_i} P_i \tag{4.38}$$

Under (4.35), this leads to

$$-1 \leq 1 - 2P_i < \nu_i < 1 \tag{4.39}$$

CHAPTER 5

SIMULATION OF THE POWER CONTROL MODEL

5.1 Effect of the Pricing Parameters

The proposed power control scheme is simulated numerically using MATLAB. First, we investigate effects of different pricing schemes for both symmetric users and multiple groups of mobiles. Then, we analyze the robustness of the system under varying parameters such as noise, the number of users, and channel gains. All results of the simulations are valid for both update schemes PUA and RUA, where the only difference between the two is the convergence rate.

Simulation parameters are chosen as follows, unless otherwise stated: spreading gain $L = 800$, noise $\sigma^2 = 10$, the stopping criterion or distance to equilibrium $\epsilon = 10^{-5}$. The channel gains of users are determined randomly with uniform distribution, $0.2 < h_i < 1$, to be more realistic. The initial condition for simulations is $p_i = 1$, $\forall i$, an estimated value for establishing initial communication between the mobile and the base station. In the simulations, a discrete time scale is used. In the delay-free case, time span is chosen long enough for perfect information flow to users. Then, delay is introduced to the system to make the setting more realistic.

In the first simulation, proportional and fixed pricing schemes are compared. For simplicity, we first choose the users being symmetric both under fixed pricing, $\lambda_i = \lambda$, and proportional pricing, $\lambda_i = kh_i$. For illustrative purposes, the number of users is chosen small, $N = 20$.

In Figure 5.1, the equilibrium power and the SIR values of each user are observed under both pricing schemes. In the top graph, power values of the users with different channel gains are almost the same under fixed pricing. Hence, the users with lower channel gains fail to meet the minimum SIR goal, chosen arbitrarily as 10 in the middle graph. In contrast, all users meet the minimum SIR level under proportional pricing, regardless of their channel gain. An intuitive explanation for this is that under proportional pricing the distant users are allowed to use more power to attain the necessary SIR. We also note that, proportional pricing is "fair" in the sense that the users are not affected by their distance to the base station.

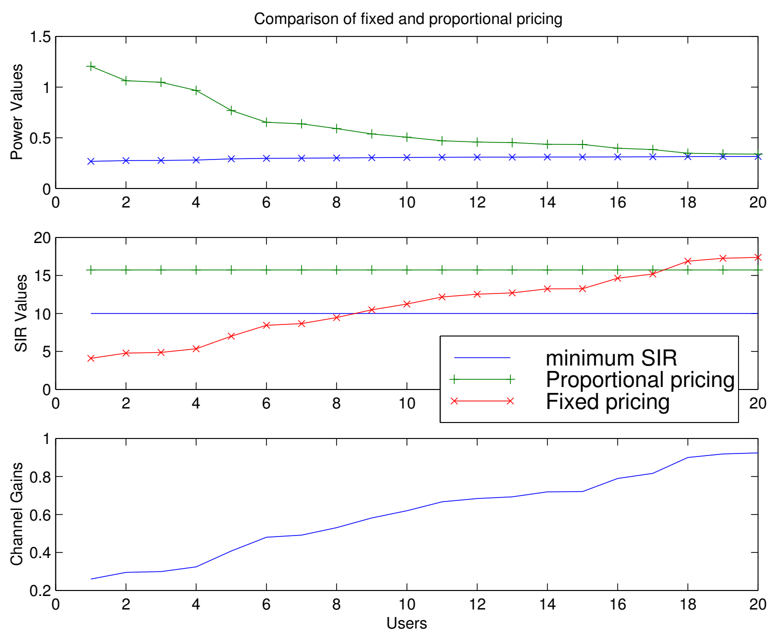


Figure 5.1 Comparison of power and SIR final values of the mobiles for the fixed and proportional pricing schemes.

The effect of varying prices is investigated in the following simulations for a single class of users by changing the pricing parameter k under proportional pricing. This parameter plays a crucial role in the system by affecting the overall power and SIR levels. From Figure 5.2, we see that a gradual increase in k from 0.5 to 6 (i.e., an increase in price)

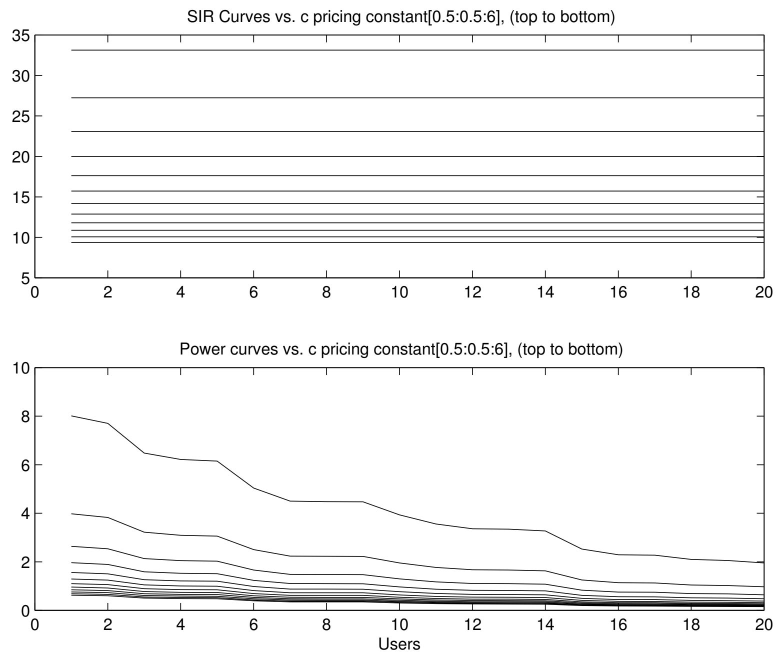
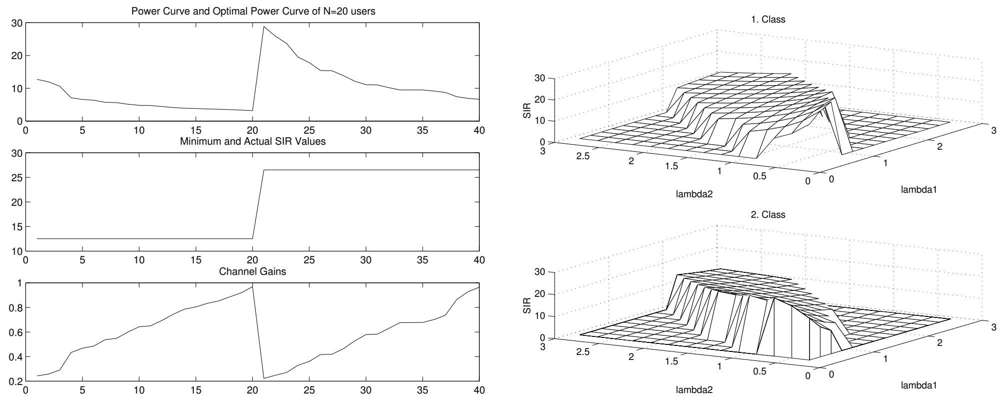


Figure 5.2 Effect of the pricing parameter k on the SIR and the power levels of users.

affects the final results in a negative way: Both power and SIR values decrease. This result might be interpreted as follows: With an increase in the price, the users decrease their powers to the same extent, which leads to lower SIR values given a constant noise level. The results match theoretical calculations for the single class case in accordance with (4.21).

As an example for multiple pricing strategies within a single cell, we have chosen the two group case for illustrative purposes. One group is given priority against the other one in terms of its SIR level. Comparable to Figure 5.1, Figure 5.3(a) shows the SIR and the power values of 40 users, with 20 in each class under proportional pricing. The user group with lower prices achieves a higher SIR, as expected. The results of this simulation justify the previous calculations in Chapter 4.

Figure 5.3(b) summarizes the SIR results for each group of mobiles under varying proportional prices: k^1 and k^2. The minimum SIR level is again chosen as 10. Parameters are zeroed out when they cannot provide this required SIR. As expected, the difference



(a) (b)

Figure 5.3 Two groups of users: (a) SIR, power, channel gains, and (b) SIR levels versus pricing parameters.

between parameters k^1 and k^2 is roughly proportional to the difference between SIR levels of the two classes. The diagonal line represents equal pricing, hence the symmetric case.

It is not always possible to achieve the requested SIR levels in a CDMA system due to inherent limitations. In such cases, the expressions (4.26)-(4.28) result in negative values, indicating that the given combination is not feasible. As a demonstration of this possibility, we calculated the range of pricing parameters k^1 and k^2 for different numbers of users in each group, N^1 and N^2. Negative parameters are nullified, indicating the boundaries of the feasible region. Figure 5.4 summarizes the outcome of the calculations graphically.

Next, we simulate users under the most general pricing scheme, where for each user there is a different fixed price for power. In this case, some users are randomly given priority as a result of pricing. In Figure 5.5, we see that some of the users fail to meet the required minimum SIR. In this pricing scheme, each user behaves as a group with a single user.

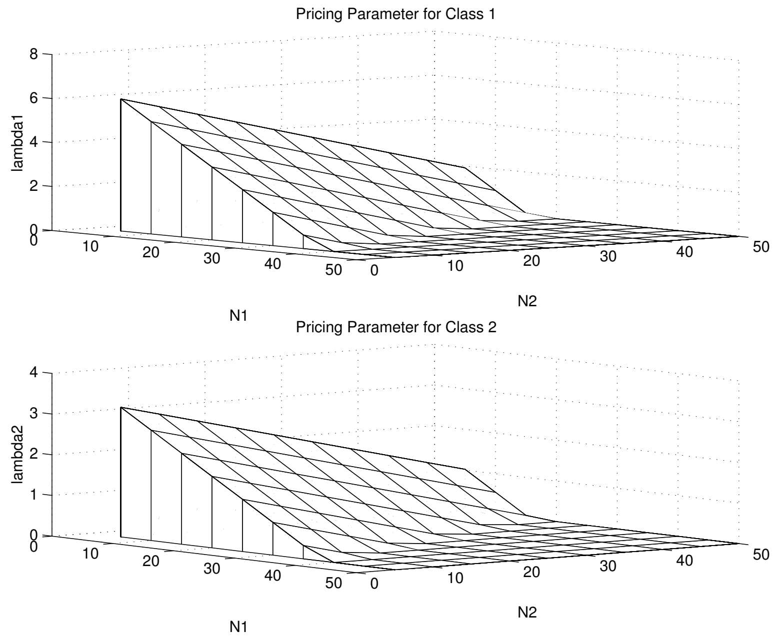


Figure 5.4 Feasible pricing parameters, k^1 and k^2, for different numbers of users, N^1 and N^2.

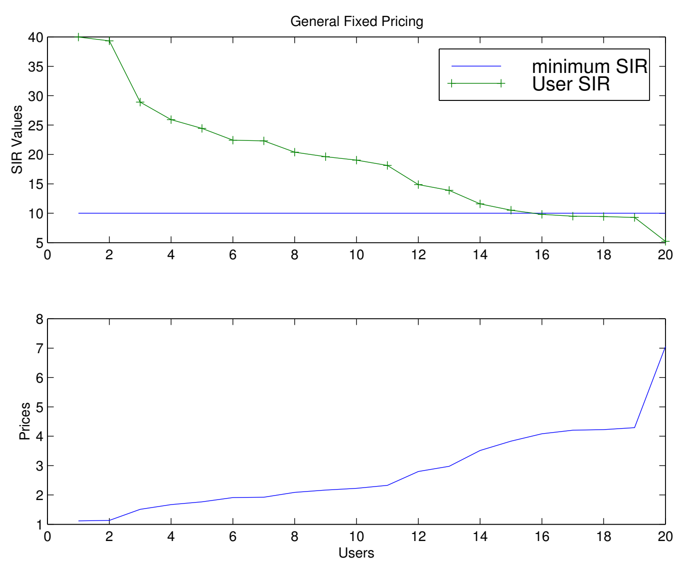


Figure 5.5 Fixed pricing scheme, general case.

5.2 Convergence Rate and Robustness of Algorithms

5.2.1 Simulations without delay

The convergence rate of the two update schemes is of great importance, as it directly affects the robustness of the system. We simulate PUA and RUA for different numbers of symmetric users under single pricing scheme. By definition, RUA with probability one is equivalent to PUA. In Figure 5.6, the number of iterations to the equilibrium point is shown for different probability values of RUA and for unit probability, PUA. Contrary to some results obtained in a different context [15], PUA is superior to RUA for this specific power game. A simple explanation for this difference can be the asymmetric nature of the users due to different channel gains.

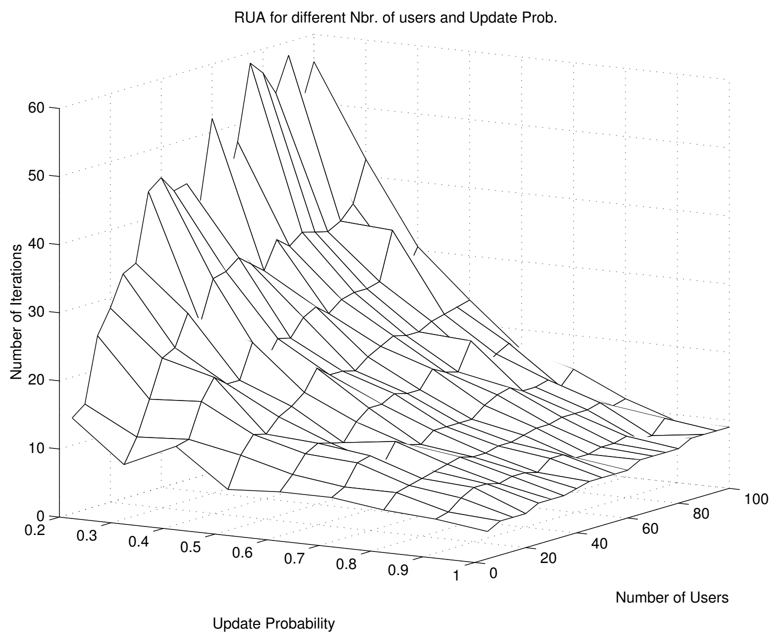
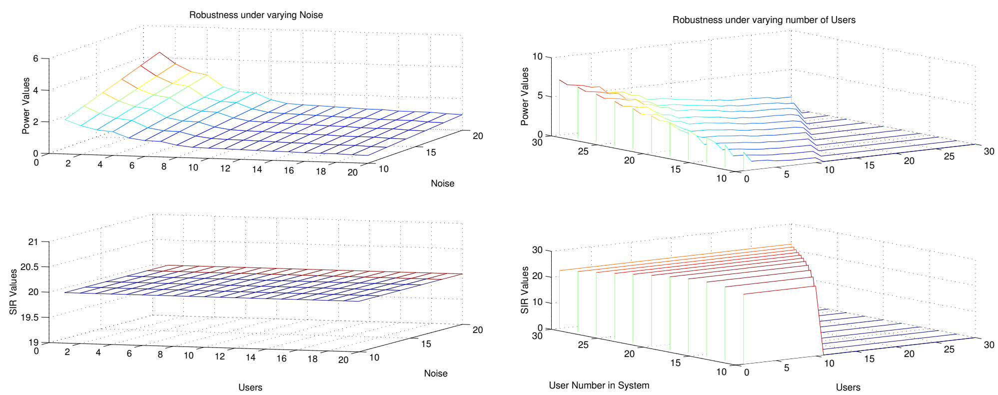


Figure 5.6 Convergence rate for different update probabilities and increasing numbers of users.

Next, we investigate the robustness of the system in the ideal, delay-free case. First, we analyze it under increasing noise, σ^2. The background noise is increased step by step up to 100% of its initial value. Accordingly, the base station allows users to increase their powers by decreasing the prices by the same percentage. The simulation is made again with $N = 20$ users under a proportional pricing scheme. We observe in Figure 5.7(a), that the power values increase in response to the increasing noise to keep the initial SIR constant. Similarly, we increase the number of mobiles in the system threefold in Figure 5.7(b). It has the same effect as increasing the noise due to the nature of CDMA. Again by adjusting the prices accordingly, all users keep their SIR levels. These results confirm the robustness of the proposed power control scheme.



(a) (b)

Figure 5.7 Power and SIR final values for increasing noise (a) and numbers of users (b).

Finally, we simulate the system in a realistic setting under a single pricing scheme: The number of users, $N = 20$ taken as default, is modeled as a Markov chain with Poisson arrival and departures. We observe the average percentage difference between the theoretical equilibrium and the current operating point of the system in terms of power values for some period of time. In the simulation, PUA is chosen as the update algorithm. The initial condition is the equilibrium point for users. In Figure 5.8, it is shown that the system operates within 1% range of ideal equilibrium points.

Heretofore, robustness of the system was investigated for static mobiles. The movements of the mobiles within the cell can be modeled by changing the channel gains randomly with time. In the next simulation, the channel gains of users are varied randomly up to 15% of their previous values. From Figure 5.9, the system again operates within 1% distance to ideal equilibrium.

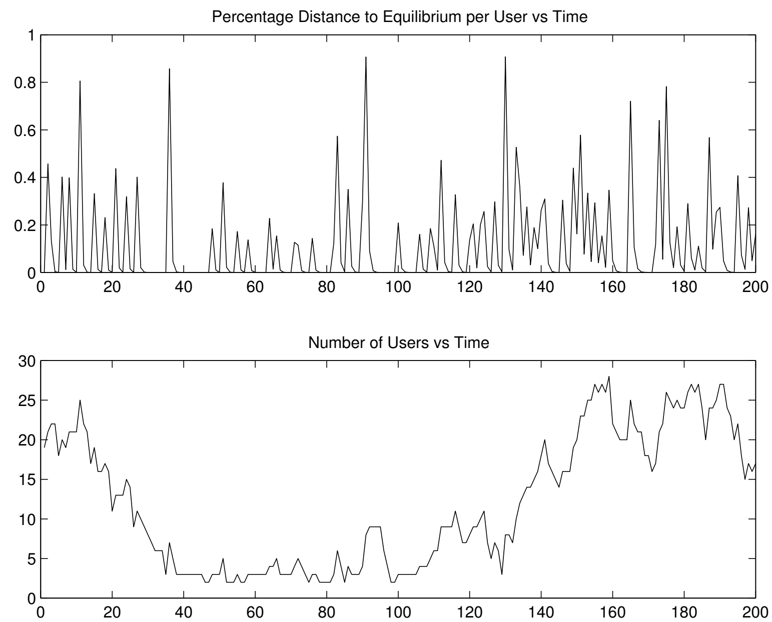


Figure 5.8 Average percentage distance to equilibrium point versus time. The number of users is modeled as a Markov chain.

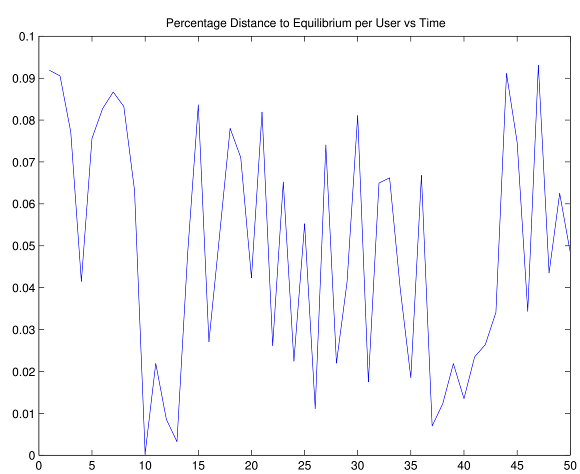


Figure 5.9 Average percentage distance to the equilibrium point versus time. Channel gains h_i are varied up to 15% per unit time.

5.2.2 Simulations with delay

We introduce the delay factor into the system in a way similar to the variable rate model: users are divided into d equal groups, where each group has an increasing number of units of delay.

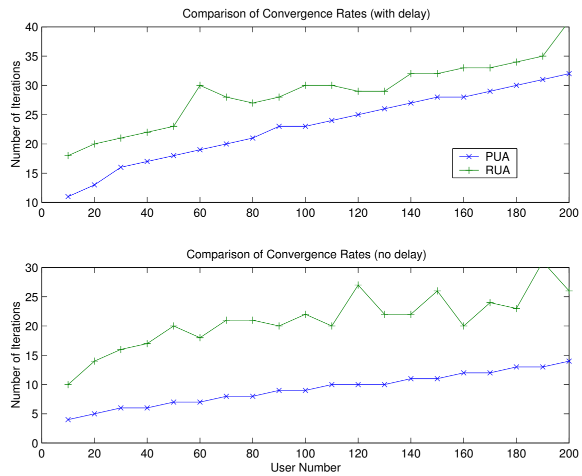


Figure 5.10 Comparison of convergence rates of PUA and RUA for increasing numbers of users in the no delay (bottom graph) and with delay (top graph) cases.

First, the convergence rates of the two update schemes are compared and contrasted under ideal and delayed conditions. The update probability of RUA is chosen as 0.5. As can be observed in Figure 5.10, there is no substantial difference between the delayed and ideal cases. In both cases RUA outperforms PUA. In the delayed case, however, the difference between the two narrows.

Then, the simulation investigating the convergence rate of RUA for various update probabilities is repeated in the delayed case. The result obtained is almost identical to the previous one shown in Figure 5.11. When this result is compared with the results of Chapter 3, it is quite surprising. A possible explanation for the discrepancy is the effect of channel gains of the mobiles, which eliminates the symmetry between the users. This effect is absent in the variable rate model.

Finally, the robustness of the PUA algorithm is investigated in the delayed case and under a multiple pricing scheme. There are two groups of users, which are symmetric

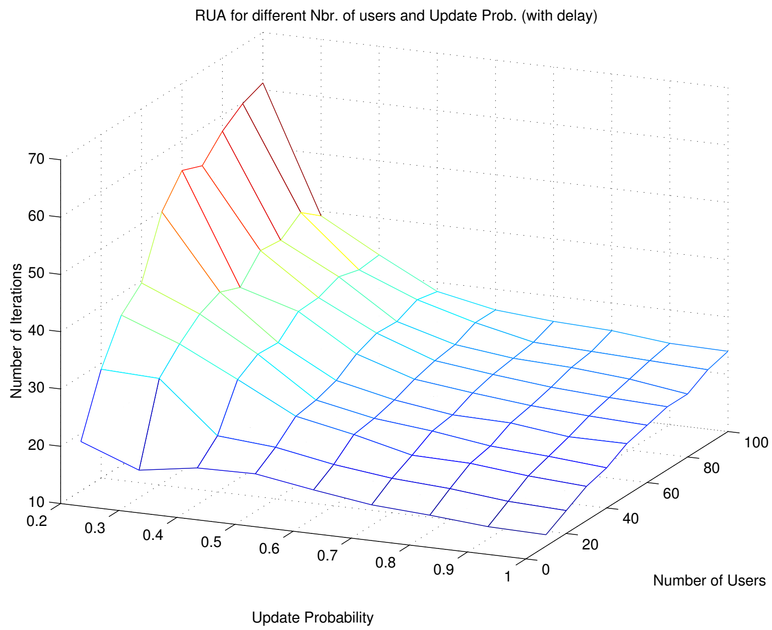


Figure 5.11 Convergence rate for different update probabilities and increasing number of users in the delayed case.

within themselves, but they are charged differently for the power they use. The parameters of the simulation are the same as those in the robustness simulation before. In Figure 5.12, it is observed that a less charged priority user always obtains a higher SIR than a regular user. The fluctuation of the SIR of both users suggests that base station should update the prices depending on the number of users in the cell.

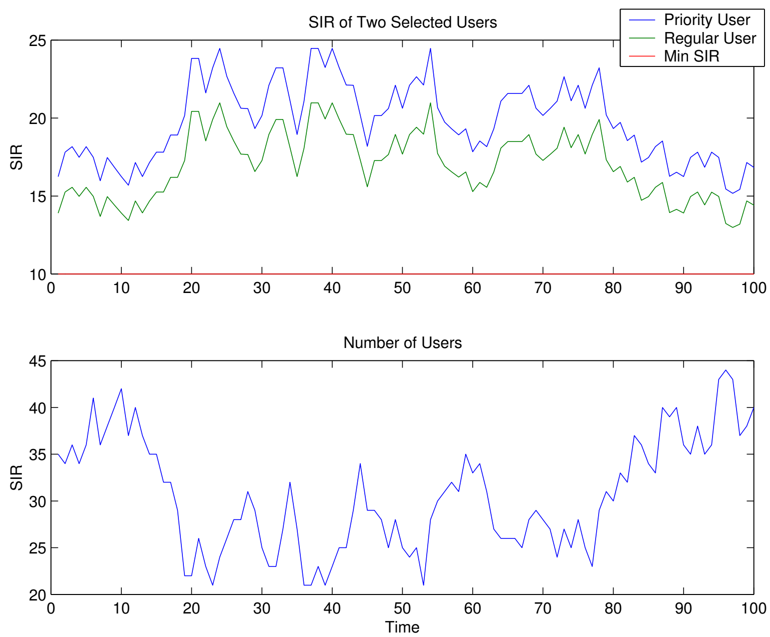


Figure 5.12 SIR of two selected users from priority and regular user groups versus time. The number of users is modeled as a Markov chain.

CHAPTER 6

CONCLUSION

We developed two mathematical models based on noncooperative game theoretical framework and obtained distributed, asynchronous control mechanisms for the two problem settings in Internet style and wireless networks. In both cases, we obtained promising results, which indicate that the game theoretical approach can provide satisfactory decentralized and market-based solutions.

While each model addresses different problems, they share some common points. A unique Nash equilibrium is proven in both of the models, and convergence properties of relevant asynchronous update schemes are investigated theoretically and numerically in each case. Moreover, conditions for the stability of the unique equilibrium point under the update algorithms are obtained and analyzed accordingly.

In the variable rate model, we have introduced an approach that can be used as a basis for implementing real time traffic on the Internet. The combination of admission control and end-to-end distributed flow control results in a very flexible framework, which captures all traffic types from low to medium elasticity. A market-based approach enables the model to address two major issues (pricing and resource allocation) simultaneously.

The simulation results suggest the use of GUA or RUA in heavy loaded systems with less delay and high demand for bandwidth. In delayed systems, however, PUA performed better than the other two. The affine utility analysis not only provided a local approximation to the nonlinear cost, but also established convergence and stability results, helping to solve the general nonlinear cases.

The second model addresses uplink power control problem in CDMA systems. We have proven that the unique Nash equilibrium has the property that, depending on the parameters, only a subset of the total number of mobiles are active. Some of the users are dropped from the system as a result of the power optimization. Furthermore, the relationship between the SIR levels of the users and the pricing is investigated for different pricing schemes. It is shown both analytically and through simulations that choosing an appropriate pricing strategy guarantees meeting the minimum desired SIR levels for the active users.

There still exist, however, some open questions on both models, which require further investigation. In the variable rate model the following questions come up, fostering directions for future research. (i) The model is analyzed here for a single bottleneck node. Possible implementations for a general network topology and routing problem are open points. (ii) The effect of varying the virtual capacity C and the initial admission scheme are possible points of investigation. In terms of pricing, the relation between fixed and variable prices should be investigated. (iii) Although the model is designed to share network resources with a best effort type distributed, elastic network like the Internet, we have not addressed possible issues regarding the interaction of different protocols on the same network. Such an interaction may be a rich source for further research. Priority queueing, for example, might be investigated, as a means for combining the protocols at the router level. In case of the power control model, one possible extension could be a multiple cells model, where the effect of neighboring cells are taken into account. Another topic of research can be the pricing strategy to be implemented for varying numbers of users in a cell.

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