Wavelet Based Subband Vector Quantization Algorithm for Gray Images

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***Abstract*- Wavelet decomposition coefficients of a gray level image are vector quantized with flexible, variable sized codewords and a wavelet based tree search algorithm. Vector quantization codewords are designed as rectangles with ratio 1 to 2, which is more efficient in encoding different subbands with minimum error. The performance of the discrete cosine transform is observed in different bands. The low frequency subband is scalar quantized due to the poor performance of DCT in this band.**

I. INTRODUCTION

Subband decomposition is a widely used and accepted technique for image compression. A typical coding scheme can be modelled as follows [1]: (1) subband decomposition; (2) quantization of subbands; (3) entropy coding.

There are several approaches for subband decomposition, zero tree and balanced tree being two widely used examples [2]. Recently, discrete wavelet transform (DWT) based decompositions have become by far the most popular alternatives for image compression purposes.

In the second stage, although there exist many scalar quantization schemes such as pulse code modulation (PCM), differential PCM (DPCM), adaptive DPCM, vector quantization (VQ) shows a better performance since its efficiency increases in proportion to the source correlation. The main drawbacks of VQ algorithms are the long search times needed for coding process and high memory requirements. Classification, tree structure, partitioning are proposed solutions for limiting the search duration in VQ.

In this paper we follow the procedure summarized above for image compression. However we introduce efficient method that improves the search algorithm of VQ by decreasing the search time and computation appreciably without significant loss of quality. We also use codewords with rectangular shape to the contrary of traditions, to exploit the correlation between wavelet coefficients in higher frequency sub-bands. For the lowest frequency subband, where the most important data are located, we use a scalar quantization scheme known as deadband quantization at the final stage. In a natural image, most of the energy is contained in the lowest frequency subband [2]. Therefore, we allocate half of our bit budget to this band.

II. WAVELET BASED VECTOR QUANTIZATION

Although VQ is the best way of quantizing and compressing images, it has a major drawback in the amount of computations during the search for optimum codevector in encoding [4, 5]. This complexity can be reduced by using an efficient codebook design and wavelet based tree structure. We take multiple stage discrete wavelet transform of codewords and use them in both search and design processes. Accordingly, our codebook consists of a table, which includes only the wavelet coefficients .

One of the search time limitation algorithms is tree structured vector quantization (TSVQ) [5, 9]. The key idea in this algorithm is finding Representative codevectors for each stage are found by first combining n codewords in k groups, where kn gives the codebook size. After obtaining the k groups, one may take the centroids of the clusters as their representative vectors. Processing as before, we can decrease the number of representative vectors. This procedure enables us to obtain correct codevectors in $2n$ comparisons for the two stage design instead of nk computations. In the design stage, obtaining the clusters plays an important role, and many times one can end up with non-optimum centroids, which results in incorrect codevector correspondences.

In order to decrease the computation time, after the standard design procedure, we can replace the representative vectors (each representing n vectors) by low bands of their wavelet transforms. An m stage TSVQ structure possesses m^n codevectors. For the kth stage, we can replace each representative vector by the lower bands of the (m-k)th wavelet transforms whichhave dimensions of $ab/2^{2(m-k-1)}$ for the codevectors with original size of ab. Further simplification can be realized during the design stage. One can start with the original codevectors and combine them into the lowest group clusters. After obtaining the centroids of the clusters, we get the last stage representatives. Then one can take the first stage wavelet transforms of the representative vectors and use only the lower bands as new representatives. Again with the same procedure, we can obtain the above stage clusters which include wavelet transforms. Proceeding similarly gives the whole structure with less computation.

III. THE ALGORITHM

The first stage subband decomposition is done through the two-dimensional wavelet transform on the image. We have chosen the Daubechies family of basis functions for DWT, which is widely used in image compression [6].

After the decomposition process, we have four different energy bands at hand: low-low (LL), low-high (LH), high-low (HL), and high-high (HH) parts. The least energy containing and the most redundant band is HH, which we have simply ignored, and actually treated as Gaussian noise [7].

The LH and HL bands also exhibit the characteristics of a high frequency signal; but there exists correlation among the horizontal and vertical pixels for the former and latter bands, respectively. Intuitively, one can try to use classical transform coding techniques, such as the discrete cosine transform (DCT) [3], for these bands. This happens to be a useless attempt due to the fact that the coefficient distribution of the resultant transformed matrix does not exhibit localization of high energy coefficients as expected in general. The main reason behind this phenomenon is the absence of high correlation between the pixels. The correlation matrices and the distribution of the DCT coefficients are shown in Figure 1 and Figure 2.

Due to the reasons stated above, one should use quantization for compressing these bands unless they will simply be dismissed as well. Usually, scalar quantization followed by entropy coding is applied to these bands as the quantization-compression scheme. However, we claim that for a variable sized codeword, VQ should give promising results when the codebook is trained effectively.

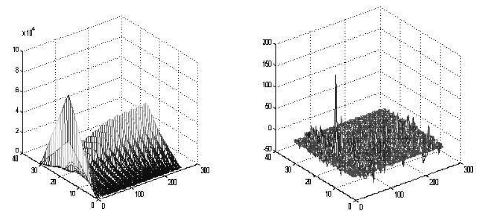

Figure 1: Autocorrelation matrices for the LL and LH bands, respectively.

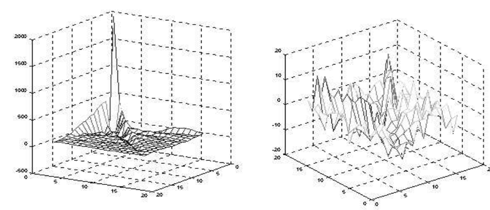

Figure 2: DCT coefficients for the LL and LH bands

At this point, one can object to this approach by the following argument: VQ is efficient over scalar quantization mainly because it exploits the high correlation in the signal [8, 9]. But it is also known that these bands are not correlated as stated before, and VQ can not produce acceptable performance. However, correlation exists in horizontal or vertical directions for these bands, and hence, by choosing the codevector sizes appropriately, we can use vector quantization most efficiently. We utilize a new construction of the codevectors in order to exploit the interpixel dependencies in these bands. In particular, we propose 4×8 blocks for the LH band, and 8×4 blocks for the HL parts in the vector quantizer code vectors.

For memory considerations, we employ the same codebook for both bands, but each codevector is also used for another band by taking the transpose of it in order to obtain the appropriate size. This saves from needing two different codebooks, which would double the memory allocation requirement of the scheme. In Figure 3 one can observe this in Lena image subbands. In the LH and HL bands of the image, we observe similar patterns which can be coded by the same codevectors.

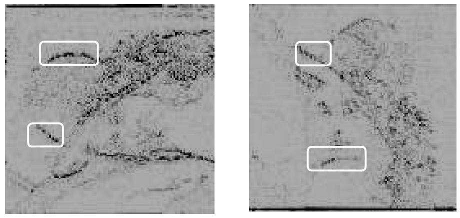

Fig. 3. Similarities between subbands

After discarding the HH band and coding the HL and LH sidebands, we further decompose the LL band. This does not bring together much computational load, because the size of the LL band is one fourth of the original image. At this point, the LL band is replaced by the following bands: low-low-low-low (LLL), low-low-high-low (LHL), low-low-high-high (LHH) and low-low-low-high (LLH). LHH band contains significant amount of energy and can not be discarded. We apply the same quantization scheme described earlier in this section to all three high frequency bands of the second level transform, namely LLH, LHL and LHH.

4×8 blocks used for the HL and LH bands are wavelet transformed to 2×4 blocks for usage in LHL, LLH and LHH. Although it may look as if we are generating a separate codebook consisting of 2×4 codevectors, the difference lies in the training and coding stages. The 4×8 and 2×4 blocks are in effect simultaneously trained since the codevectors are related through the wavelet transform. The only additional computation is in the training stage, where the DWT of the codevectors are calculated.

After deciding on the codebook size, one has to determine the VQ search algorithm. In this paper, we employed the wavelet tree structured vector quantization (WTSVQ), where during the construction of the tree structure, the vectors are combined according to the Euclidean distance between the LL bands of their wavelet transforms. Thus, the LL band of the codevector becomes the representative for that vector. In fact, WTSVQ can be interpreted as a modified version of the classified VQ scheme. This approach reduces the computational complexity as we work with $m/2 \times n/2$ vectors rather than $m \times n$.

One other advantage of WTSVQ is its capability of decreasing the codevector dimensions. As a result, when clustering the vectors, the reduction in the dimension of the vector space enables very effective clustering, which leads to more efficient trees. The codebook is trained over this structure with more pictures in a shorter time period.

The tree is constructed in two stages. In the first stage we have p representative codewords. Below this layer, we have r additional codevectors related to each of the p first stage codevectors, giving rise to a codebook of size rp. After the tree structure is set up, we begin to use the elements of the training set to develop the codebook. We take the first subblock of the training set matrix, and compare it to the p first stage elements, obtaining an index of the best-fitting element. Then we compare the training set subvector with the r elements which lie below the first stage's best fitting vector. This process requires $r+p$ comparisons for determining the best fitting codeword instead of rp.

Last part of the compression scheme involves the coding of the LLL band. Although there is high correlation among the pixels of this band, the DCT coefficients are not localized enough. From Figure 2, we observe that there is a peak in the upper left part of the DCT matrix, as well as deep fluctuations in the diagonal elements of this matrix. Thus it is not possible to discard as many DCT coefficients as it is usually done for natural images, and scalar quantization is used for this band. In fact, for further compression quantized coefficients can be entropy coded. Since we are mainly concerned with VQ and its use in subband coding, we do not consider scalar entropy coding in this paper. However we implement a deadband scalar quantization for the LLL subband, where all higher frequency bands of natural images exhibit Gaussian distribution in their histograms. Therefore if the coefficients close to zero are discarded, a good compression ratio can be achieved.

IV. RESULTS AND CONCLUSION

In the proposed algorithm, a single codebook is used for all subbands. We used the Lena image (256×256) in the experiments due to its very good histogram distribution. For Lena, the number of common indices of codevectors among high frequency bands is displayed in Table 1.

	LH	HL	LHH	LLH	LHL
LH	***512***	131	91	92	53
HL	131	***512***	95	111	76
LHH	91	95	***512***	142	118
LLH	92	111	142	***512***	118
LHL	53	76	118	118	***512***

Table 1: The number of common codewords for the Lena image

Table-1 indicates that there exists a significant amount of correlation between the subbands. Between some bands, as many as 20-25% of the indices are in common. We observe that there is more than 20% correlation between the bands at each level; for example, LH versus HL, and LHL versus LLH. There

is also some correlation between LHH, LHL and LLH bands. This evidence is in support of using the same codebook for different bands by just changing the codevector dimensions. There is also more than 10% common sharing of codevectors between bands of different levels due to the codebook structure.

The compressed Lena pictures are shown in Figures 4 and 5 for two different codebooks trained with three and ten images, respectively. The compression ratio is 12:1 with approximately 0.6 bits per pixel on the average.

Figure 4: Lena image with wavelet based subband VQ trained over three images, with compression ratio 12:1.

Figure 5: Lena image with wavelet based subband VQ trained over ten images, with compression ratio 12:1.

In Table 2, the image quality offered by the proposed compression algorithm is evaluated against the JPEG standard in terms of the peak signal to noise ratio (PSNR), the average difference (AD) and the image fidelity (IF) [10]. Notice that we have worked with 256×256 images, which resulted in PSNR ratios lower than usual.

	PSNR	AD	IF
3 Image Training	25.979	6.11e-05	0.9868
10 Image Training	27.042	6.11e-05	0.9895
JPEG	28.511	0.0220	0.9927

Table 2: Performance evaluation of the images using various quality measures.

The results in Table 2 justify that with a better trained codebook, the performance of the proposed algorithm approaches JPEG for both PSNR and IF measures, whereas in terms of the AD criterion it outperforms the standard competition.

The final conclusion is that DCT based algorithms, which were considered to ensure higher fidelity turned out to be inefficient.

V. References

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