A Hybrid Game Approach for Joint User Selection and Power Allocation over Unlicensed Spectrum Bands

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***Abstract*— This paper studies sharing of unlicensed spectrum bands between network operators by formulating it as a hybrid noncooperative game. A multi-user multi-channel downlink transmission is considered where the communication links are modeled as Gaussian parallel interference channels. Each transmitter, controlled by a competing operator, selects one user per channel from its registered users as well as the transmission power level on each channel.**

The operators are selfish and aim to maximize their sum-rate which is captured by a hybrid noncooperative game model. Nash equilbrium provides an incentive-compatible and suitable solution concept for this game. Sufficient conditions for the existence and uniqueness of a (pure) Nash equilibrium are derived. Furthermore, an algorithm is proposed to compute all Nash equilibria. Additional insights into user selection in the high SNR and low SNR regimes are developed via a numerical example. Finally, a mechanism based on pricing is investigated to drive the Nash Equilibrium of the game to a pareto-optimal solution, for example, by an independent regulator.

I. Introduction

As the need for and importance of wireless communication increases, there has been a growing interest in using the available and limited spectrum. Cognitive radios and opportunistic use of television white space spectrum are two paradigms that demonstrate this trend. Another related development is the promotion of market-based mechanisms for more efficient sharing of wireless spectrum by promoting a competitive environment among operators.

This paper focuses on the problem of co-existence of multiple, selfish, and competitive operators in an unlicensed band. How should each operator use its available resources in such an environment? We study this problem by formulating a noncooperative game between several operators. Each operator has one transmitter that communicates to its own set of registered users over a downlink such that a given user is served by only its own operator. Motivated by the physical layer considerations (e.g. in IEEE 802.22 specifications), we model the underlying communication link as a set of parallel and independent interference channels. On each channel, one user for each operator is served. Furthermore, each transmitter has a total power constraint and needs to allocate power across all available channels while maximizing the communication rate. Thus, the decision variable in the game is a joint selection of users and powers on each channel making it a hybrid game.

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The Nash equilbrium provides an incentive-compatible and suitable solution concept for the game formulated. We derive sufficient conditions for the existence and uniqueness of a (pure) Nash equilibrium. Furthermore, an algorithm is proposed to compute all Nash equilibria. Additional insights into user selection in the high SNR and low SNR regimes are developed via a numerical example. Finally, a mechanism based on pricing is investigated to drive the Nash Equilibrium of the game to a pareto-optimal solution, for example, by an independent regulator.

The rest of the paper is organized as follows. The next subsection gives an overview of background on spectrum sharing and related work. We introduce the channel model and the underlying game formally in Section II. We identify the game as a hybrid game and propose an algorithm to characterize all the Nash equilibrium solutions in Section III. Some insights into selection between *interference limited* and *noise limited* users are developed in Section IV via a numerical example. Section V introduces a simple distributed algorithm for the joint user-selection and power-allocation problem. Finally, Section VI suggests an approach to drive the Nash Equilibrium to any desired operating point through a judicious choice of pricing mechanisms.

A. Background and Related Work

The recent report and order issued by the Federal Communications Commission (FCC) in the United

States permits use of the television (TV) white space spectrum [1] by devices equipped with cognitive radio. These rules have created new opportunities for developing wireless networks in frequency bands, which have superior propagation and building penetration than the ones used by traditional networks (e.g., WiFi, Bluetooth etc.). On the other hand, this newly available spectrum varies with location, there are strict restrictions on tranmission power, and devices using it need to ensure that they do nto cause interference to a *primary* device operating in the same spectrum [2].

A number of standardization activities have emerged to identify key challenges and develop new solutions for using the TV white space spectrum with cognitive radios. One such activity is conducted by the IEEE 802.22 Wireless Regional area network (WRAN) [3]. The physical (PHY) layer in this standard uses orthogonal frequency division multiple access (OFDMA) based modulation schemes for both uplink and downlink transmission [4]. OFDMA is a natural choice since at any location it treats the available multiple disjoint TV bands (each of 6MHz bandwidth) as a sequence of parallel channels. Hence, the need for contiguous spectrum is avoided.

In related works, the problem of power allocation over interference channels has been studied when there is one receiver for each transmitter. An iterative water-filling strategy for power allocation has been proposed in [5] and further developed in [6], [7] and references therein. Sufficient conditions on the channel gains for the convergence of the iterative algorithm have been derived in these references. For a game theoretic treatment of the power-allocation problem see [8], [9]. Finally, some pricing mechanisms to improve the Nash Equilibrium operating point have been discussed in [10].

In contrast to existing literature we consider a point-to-multipoint scenario, where each transmitter serves multiple receivers. The transmitter not only selects the transmit power on each channel but also selects a user on each channel. Furthermore, each transmitter is controlled by a single selfish and competitive operator. We formulate this problem as a hybrid noncooperative game and propose an algorithm to determine all the (pure) Nash equilibrium points of this game. Furthermore, we develop some insights into how the user selection can vary in the low and high SNR regimes.

II. Channel Model

We consider a scenario with $N = 2$ operators, K users per operator and M parallel channels. The channel model between operator n and user $k \in \{1,2,\dots,K\}$ is

$$y_{nk}(m) = \sqrt{h_{nk}(m)}x_k(m) + \sqrt{g_{jk}(m)}x_j(m) + z_k(m),$$
$$m = 1,2,\dots,M, \quad n,j = 1,2, \quad j \neq n. \quad (1)$$

Here $h_{nk}(m)$ denotes the direct channel gain between the transmitter of operator n and its user k on channel m and $g_{jk}(m)$ denotes the interfering gain between the transmitter of operator j and the user k of the other operator and on channel m. The additive noise $z_j(m)$ is assumed to be Gaussian and independent for each channel. Without loss in generality, we assume that it is of unit variance. Furthemore, we note that while our model restricts to the case of two operators for simplicity, the results easily extend to an arbitrary number of operators. Although we only consider real-valued channel gains, we can also extend the results to complex valued channel gains, by treating the real and imaginary parts separately.

A. Decision variables and Utility function

Each operator has a total power constraint $\bar{P}$, which is split across the M sub-channels. The power of operator n on channel m is denoted by $p_n(m)$ and the set of all feasible power allocations is given by

$$\mathscr{P}_n = \left\{ \mathbf{p}_n \in \mathbb{R}^M \;\middle|\; p_n(m) \geq 0, \sum_{m=1}^{M} p_n(m) \leq P \right\}, \quad (2)$$

where we denote the power vector selected by operator n by

$$\mathbf{p}_n := (p_n(1), \dots, p_n(M)).$$

For each channel $1 \leq m \leq M$, the operator $n \in \{1,2\}$ selects user $k_n(m)$ from the set of users $\{1,2,\dots,K\}$ subscribed to it. We denote the set of all users selected by operator n by $\mathbf{k}_n$. The utility function for operator n, given the choice of power levels $\mathbf{p}_n$ and users $\mathbf{k}_n$ is given by the corresponding sum-rate achieved using Gaussian codebooks and treating interference as noise [11],

$$U_n(\mathbf{p}_n, \mathbf{k}_n; \mathbf{p}_j) = \sum_{m=1}^{M} \frac{1}{2} \log \left(1 + \frac{p_n(m) h_{nk_n(m)}(m)}{1 + p_j(m) g_{jk_n(m)}(m)} \right),$$
$$n, j = 1,2, j \neq n. \quad (3)$$

Each selfish operator tries to maximize its own utility given the actions of others resulting in a interference-coupled and competitive environment.

B. Nash equilibrium solution

Nash equilibrium provides a useful solution concept for noncooperative games. At the Nash equilibrium solution, no player has any incentive to deviate from it. We say that $(\{\mathbf{k}_1^\star, \mathbf{p}_1^\star\}, \{\mathbf{k}_2^\star, \mathbf{p}_2^\star\})$ is a Nash equilibrium (NE) solution to the joint user-selection and power-allocation game if

$$\begin{aligned} U_1(\mathbf{p}_1^\star, \mathbf{k}_1^\star; \mathbf{p}_2^\star) &\geq U_1(\mathbf{p}_1, \mathbf{k}_1; \mathbf{p}_2^\star), \\ U_2(\mathbf{p}_2^\star, \mathbf{k}_2^\star; \mathbf{p}_1^\star) &\geq U_2(\mathbf{p}_2, \mathbf{k}_2; \mathbf{p}_1^\star), \end{aligned} \tag{4}$$

for all feasible power vectors $\mathbf{p}_1 \in \mathscr{P}_1$, $\mathbf{p}_2 \in \mathscr{P}_2$ and user-selection vectors $\mathbf{k}_1 \in \{1, \ldots, K\}^M$ and $\mathbf{k}_2 \in \{1, \ldots, K\}^M$. The Nash equilibrium is unique if there is a unique solution to (4).

We note that two separate NE are discussed in the rest of the paper: one of the power-allocation game where the user selection is pre-determined and one of the joint user-selection and power-allocation game. We identify each with the specific game mentioned.

III. Hybrid Game Approach

The joint user-selection and power-allocation game is a hybrid game where each player selects users from a discrete set and power levels from a continuous set. To obtain a pure NE solution to this game we exploit the hybrid structure and introduce an order in selection as described below.

1) For each choice of user selection vectors $(\mathbf{k}_1, \mathbf{k}_2) \in \{1, \ldots, K\}^M \times \{1, \ldots, K\}^M$ we compute the Nash Equilibrium power-allocation vectors using the iterative water-filling algorithm in [5], [7]. Fig. 1 shows the set of power allocation vectors for each choice of users for $K = 2$ users and $M = 2$ channels. If the channel satisfy the sufficient condition in Lemma 1 the iterative water-filling algorithm converges to a unique solution $(\mathbf{p}_1^\star, \mathbf{p}_2^\star)$ for each pair of users $(\mathbf{k}_1^\star, \mathbf{k}_2^\star)$.
2) For each pair $(\mathbf{k}_1^\star, \mathbf{k}_2^\star)$ and for each $\mathbf{k}_1 \in \{1, \ldots, K\}^M$ we consider a deviation $\mathbf{k}_1^\star \to \mathbf{k}_1$ and compute the resulting power vector for this deviation

$$\mathbf{p}_1(\mathbf{k}_1; \mathbf{k}_1^\star, \mathbf{k}_2^\star) = \arg \max_{\mathbf{p}_1 \in \mathscr{P}_1} U_1(\mathbf{p}_1, \mathbf{k}_1; \mathbf{p}_2^\star) \tag{5}$$

Likewise, for each deviation $\mathbf{k}_2^\star \to \mathbf{k}_2$, we compute the resulting power vector

$$\mathbf{p}_2(\mathbf{k}_2; \mathbf{k}_1^\star, \mathbf{k}_2^\star) = \arg \max_{\mathbf{p}_2 \in \mathscr{P}_2} U_2(\mathbf{p}_2, \mathbf{k}_2; \mathbf{p}_1^\star) \tag{6}$$

3) We say that $(\{\mathbf{k}_1^\star, \mathbf{p}_1^\star\}, \{\mathbf{k}_2^\star, \mathbf{p}_2^\star\})$ is a *dominant pair* if the following is satisfied:
   - For each $\mathbf{k}_1 \in \{1, \ldots, K\}^M$, if $\mathbf{k}_1 \neq \mathbf{k}_1^\star$ we have that
   $$U_1(\mathbf{p}_1^\star, \mathbf{k}_1^\star; \mathbf{p}_2^\star) > U_1(\mathbf{p}_1(\mathbf{k}_1; \mathbf{k}_1^\star, \mathbf{k}_2^\star), \mathbf{k}_1; \mathbf{p}_2^\star) \tag{7}$$
   - for each $\mathbf{k}_2 \in \{1, \ldots, K\}^M$, if $\mathbf{k}_2 \neq \mathbf{k}_2^\star$ we have that
   $$U_2(\mathbf{p}_2^\star, \mathbf{k}_2^\star; \mathbf{p}_1^\star) > U_1(\mathbf{p}_2(\mathbf{k}_2; \mathbf{k}_1^\star, \mathbf{k}_2^\star), \mathbf{k}_2; \mathbf{p}_1^\star) \tag{8}$$

The following theorem establishes connections between a dominant pair $(\mathbf{k}_1^\star, \mathbf{k}_2^\star)$ and the NE of the joint user selection and power allocation game.

Theorem 1: Suppose that for each pair of user selection vectors the power-allocation game has a unique NE i.e., step 1 above is well defined. Then, if $(\mathbf{k}_1^\star, \mathbf{k}_2^\star)$ that satisfy (7)-(8) the corresponding pair $(\{\mathbf{k}_1^\star, \mathbf{p}_1^\star\}, \{\mathbf{k}_2^\star, \mathbf{p}_2^\star\})$ is a NE in the joint user-selection and power-allocation pair. Conversely, if $(\{\mathbf{k}_1^\star, \mathbf{p}_1^\star\}, \{\mathbf{k}_2^\star, \mathbf{p}_2^\star\})$ is a NE in the joint user-selection and power-allocation pair, then they satisfy (7)-(8).

Proof: To establish the direct part, we first show that

$$U_1(\mathbf{p}_1^\star, \mathbf{k}_1^\star; \mathbf{p}_2^\star) > U_1(\mathbf{p}_1, \mathbf{k}_1; \mathbf{p}_2^\star),$$

as stated in (4). This follows since from (7)

$$\begin{aligned} U_1(\mathbf{p}_1^\star, \mathbf{k}_1^\star; \mathbf{p}_2^\star) &> U_1(\mathbf{p}_1(\mathbf{k}_1; \mathbf{k}_1^\star, \mathbf{k}_2^\star), \mathbf{k}_1; \mathbf{p}_2^\star) \\ &\geq U_1(\mathbf{p}_1, \mathbf{k}_1; \mathbf{p}_2^\star), \end{aligned}$$

where the last relation follows via (5). Similarly we can establish that

$$U_2(\mathbf{p}_2^\star, \mathbf{k}_2^\star; \mathbf{p}_1^\star) > U_2(\mathbf{p}_2, \mathbf{k}_2; \mathbf{p}_1^\star),$$

for all feasible $\mathbf{p}_2 \in \mathscr{P}_2$ and all $\mathbf{k}_2 \neq \mathbf{k}_2^\star$. To establish the the converse part, we show that if $(\{\mathbf{k}_1^\star, \mathbf{p}_1^\star\}, \{\mathbf{k}_2^\star, \mathbf{p}_2^\star\})$ is a NE solution to the original game then the resulting vectors $(\mathbf{k}_1^\star, \mathbf{k}_2^\star)$ satisfy (7)-(6). Since

$$U_1(\mathbf{p}_1^\star, \mathbf{k}_1^\star; \mathbf{p}_2^\star) > U_1(\mathbf{p}_1, \mathbf{k}_1; \mathbf{p}_2^\star),$$

it follows that

$$\begin{aligned} U_1(\mathbf{p}_1^\star, \mathbf{k}_1^\star; \mathbf{p}_2^\star) &> \max_{\mathbf{p}_1 \in \mathscr{P}_1} U_1(\mathbf{p}_1, \mathbf{k}_1; \mathbf{p}_2^\star), \\ &= U_1(\mathbf{p}_1(\mathbf{k}_1; \mathbf{k}_1^\star, \mathbf{k}_2^\star), \mathbf{k}_1; \mathbf{p}_2^\star), \end{aligned}$$

where the last relation follows from (5). This establishes (7). The relation (8) can be similarly established. ■

It remains to establish a set of sufficient conditions on the channel gains for the existence of a unique NE to the power allocation game for each for each pair of user selection vectors.

OP. 2/*OP.* 1	(1,1)	(1,2)	(2,1)	(2,2)
(1,1)	$\mathbf{p}_1(11,11),\mathbf{p}_2(11,11)$	$\mathbf{p}_1(11,12),\mathbf{p}_2(11,12)$	$\mathbf{p}_1(11,21),\mathbf{p}_2(11,21)$	$\mathbf{p}_1(11,22),\mathbf{p}_2(11,22)$
(1,2)	$\mathbf{p}_1(12,11),\mathbf{p}_2(12,11)$	$\mathbf{p}_1(12,12),\mathbf{p}_2(12,12)$	$\mathbf{p}_1(12,21),\mathbf{p}_2(12,21)$	$\mathbf{p}_1(12,22),\mathbf{p}_2(12,22)$
(2,1)	$\mathbf{p}_1(21,11),\mathbf{p}_2(21,11)$	$\mathbf{p}_1(21,12),\mathbf{p}_2(21,12)$	$\mathbf{p}_1(21,21),\mathbf{p}_2(21,21)$	$\mathbf{p}_1(21,22),\mathbf{p}_2(21,22)$
(2,2)	$\mathbf{p}_1(22,11),\mathbf{p}_2(22,11)$	$\mathbf{p}_1(22,12),\mathbf{p}_2(22,12)$	$\mathbf{p}_1(22,21),\mathbf{p}_2(22,21)$	$\mathbf{p}_1(22,22),\mathbf{p}_2(22,22)$

Fig. 1. A matrix showing power allocation vector pairs for each selection of users by operators (OP) 1 and 2. Each pair of power vectors is computed using an iterative water-filling algorithm. If the channels satisfy the sufficient condition in Lemma 1 the iterative water-filling algorithm converges to a unique solution for each pair of users.

Lemma 1: A sufficient condition for the existence of a unique NE solution to the power allocation game for each pair of user selection vectors in Step 1 of our algorithm is that

$$\max_{k\in\{1,\ldots,K\}}\ \max_{\substack{j,n\in\{1,2\},\\ j\neq n}}\ \max_{1\le m\le M}\frac{g_{jk}(m)}{h_{nk}(m)}<1. \tag{9}$$

Proof: For a given *cell c* in Table 1 above let the user selection vectors be $(\mathbf{k}_1^c,\mathbf{k}_2^c)$. A sufficient condition for the existence of a unique power allocation vector is [5]

$$\max_{\substack{j,n\in\{1,2\},\\ j\neq n}}\ \max_{1\le m\le M}\frac{g_{j\mathbf{k}_n^c(m)}(m)}{h_{n\mathbf{k}_n^c(m)}(m)}<1. \tag{10}$$

Since we need to satisfy this relation for all user selection vectors, we obtain

$$\max_{c}\ \max_{\substack{j,n\in\{1,2\},\\ j\neq n}}\ \max_{1\le m\le M}\frac{g_{j\mathbf{k}_n^c(m)}(m)}{h_{n\mathbf{k}_n^c(m)}(m)}<1. \tag{11}$$

Since the index c cycles through all possible user selection vectors the above condition is equivalent to (9). ■

Intuitively Lemma 1 states that a unique NE exists to the power allocation game if for each user in the system the direct channel gain with respect to its own operator exceeds the cross gain with respect to the interfering operator.

IV. Numerical Example

In this section we develop some insights into the interplay between user-selection and power levels of the two operators. For simplicity we consider the case when there is only a single channel i.e., $M=1$ and there are two users $K=2$. Furthermore, we assume that the users on the two links are symmetric i.e., (c.f. (1)) we have $h_{1j}=h_{2j}$ and $g_{1j}=g_{2j}$ for $j=1,2$. The specific values for channel gains of the two users are chosen as

	user 1	user 2
direct gain (h_{ij})	1	10
cross gain (g_{ij})	0	5

The utility functions (3) for the two users are given as follows

$$U_j(p,\ 1;q)=\frac{1}{2}\log(1+p),\quad j=1,2 \tag{12a}$$

$$U_j(p,\ 2;q)=\frac{1}{2}\log\left(1+\frac{10p}{1+5q}\right),\quad j=1,2. \tag{12b}$$

Intuitively user 1 is noise limited user. It does not suffer from any interference, however its cross gain is low.

We compute the NE assuming that both the operators have the same power constraint of P. We note for the NE each operator uses all the available power since regardless of the choice of the users, deviating from this point (when the other user uses all the available power) will strictly reduce the utility functions (12a), (12b). The NE utility for each users is given by

$$U_j^\star(P)=\max\left\{U_j(P,\ 1;\ P),U_j(P,\ 2;\ P)\right\} \tag{13}$$

i.e., each access point selects a user that maximizes the resulting rate. Fig. 2plots the utility functions (12a), and (12b) as a function of P. We note that in the low power regime it is favorable to select user 2 as its utility function is higher. This is explained by noting that when $P\approx 0$, we have that

$$U_j(P,\ 2;P)=\frac{1}{2}\log\left(1+\frac{10P}{1+5P}\right)\approx\frac{1}{2}\log(1+10P)$$

which exceeds $U_j(P,\ 1;P)=\frac{1}{2}\log(1+P)$. As we increase the value of P the interference produced at user 1 increases and its utility function decreases. When P is sufficiently high, it is favorable to select user 1 since for $P\gg 1$, we have that

$$U_j(P,\ 2;P)\approx\frac{1}{2}\log\left(1+\frac{10P}{5P}\right)\approx\frac{1}{2}\log 3$$

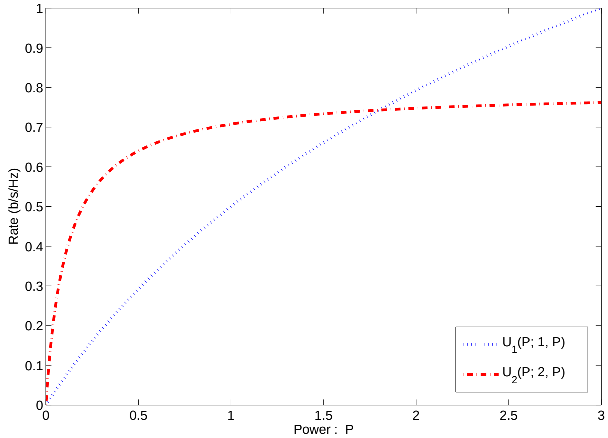

Fig. 2. The plot of utility functions given each user as a function of the maximum transmit power. The dotted curve shows utility function of user 1 (12a) while the dashed curve shows the utility function of user 2 (12b). Note that user 1 is a "noise limited" while user 2 is "interference limited" user. In the low SNR regime, the utility function of user 2 is larger than user 1 and each access point selects user 2. In the high SNR regime, the utility function of user 1 is larger and each access point selects user 1.

whereas the rate when selecting user 1

$$U_j(P,\ 1;P) = \frac{1}{2}\log(1+P)$$

increases with unboundedly with the transmit power.

Fig. 2 indicates that for the specific choice of channel gains in this examples, the operators select user 2 at the NE when $P \lessapprox 1.8$ and select user 1 otherwise. More generally the NE favors selecting interference limited users in the low-SNR regime while noise limited users in the high-SNR regime.

V. Iterative User Selection and Water-Filling Algorithm

Our discussion so far focusses on the existence of Nash Equilibrium. In practice it is desirable to have a distributed algorithm that can be implemented independently by each operator. Such algorithms have been proposed for the case of a single receiver in e.g., [5], [7]. In this section we generalize these algorithms to include the user selection step. In what follows we assume that there are K users for each access point, M parallel interference channels and one transmitter for each of the N operators.

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1) Initialize the power levels

$$\mathbf{p}_n^\star = [P/M, P/M, \dots, P/M]$$

2) repeat the loop below for T steps
   - for each access point $n = 1, 2, \dots, N$
   - for each channel $m = 1, 2 \dots, M$
   - for each $k = 1, 2, \dots, K$, compute

     $$SINR_{nk}(m) = \frac{P_n(m)h_{nk}(m)}{1+\sum_{j\neq n} g_{jk}(m)P_j(m)}$$

     the signal-to-interference plus noise ratio of user k belonging to access point n on channel m.
   - Select $k^\star(n,m)$ — the user with the largest SINR on each channel.

     $$k^\star(n,m) = \arg\max_k SINR_{nk}(m)$$

   - Let

     $$\mathbf{k}_n^\star = [k^\star(n,1), k^\star(n,2), \dots, k^\star(n,M)]$$

     be the user selection vector for access point n.
   - update the power vector $\mathbf{p}_n$ as

     $$\mathbf{p}_n^\star = \arg\max_{\mathbf{p}\in\mathscr{P}} U_n(\mathbf{p}, \mathbf{k}_n^\star; \{\mathbf{p}_t^\star\}_{t\neq n})$$

     by standard water-filing algorithm.

. . . .

Unlike the traditional single-user iterative water-filling algorithms, the proposed algorithm involves a user selection step from a discrete set of candidates. Hence its convergence analysis appears involved and is left for future work.

VI. Pricing Mechanisms

In general it is well-known that a Nash Equilibrium solution may not be Pareto optimal. One approach to drive the NE to a desired operating point is to introduce a pricing mechanism [12], [10]. Here we extend this approach for the joint user selection and power allocation game. Furthermore in this section we assume that there is no sum-power constraint i.e., the feasible set of power is simply

$$\mathscr{P}_n = \left\{ (p_n(1), \dots, p_n(M)) \;\middle|\; p_n(m) \geq 0 \right\}. \tag{14}$$

Suppose that it is desired that the transmitter n choose a power allocation level $\bar{\mathbf{p}}_n$ for $n = 1, 2$. In some cases, the desirable operating point of the system may be not a single point but a region which brings additional flexibility. We consider a regulator imposing a pricing mechanism to drive the NE to the desired operating point. More specifically if the transmitter selects a power level $\mathbf{p}_n$ the regulator will charge a value of $\sum_m \lambda_m \mathbf{p}_n(m)$ to the operator so that the resulting utility function becomes

$$\bar{U}_n(\mathbf{p}_n, \mathbf{k}_n; \{\mathbf{p}_j\}_{j\neq n}) = U_n(\mathbf{p}_n, \mathbf{k}_n; \{\mathbf{p}_j\}_{j\neq n}) - \sum_m \lambda_m \mathbf{p}_n(m).$$

In what follows we consider two pricing mechanisms as discussed below.

A. User dependent prices

In this case we allow the regulator to publish a separate price for each choice of user i.e., if transmitter n selects user k on channel m, the price to be paid to the regulator is $\lambda_{nk}(m)$. In order to ensure that the power used by the operator is the desired value $\bar{\mathbf{p}}_n(m)$ by following [12], we need that for each $k = 1, \ldots, K$

$$\bar{\mathbf{p}}_n(m) = \arg\max_p \ \frac{1}{2}\log\left(1 + \frac{h_{nk}(m)p}{1 + \sum_{j\neq n} \bar{\mathbf{p}}_j(m) g_{jk}(m)}\right) - \lambda_{nk}(m)p$$

which implies that

$$\lambda_{nk}(m) = \frac{h_{nk}(m)}{2(1 + \bar{\mathbf{p}}_n(m)h_{nk}(m) + \sum_{j\neq n} \bar{\mathbf{p}}_j(m) g_{jk}(m))}.$$

B. User independent prices

In general it may be prohibitively complex if the regulator is required to publish a separate price for each uses partly also to information constraints. It is natural to ask whether we drive the system to a desired operating point $\{\bar{\mathbf{p}}_n\}$ by restricting the regulator to charge a single price that does not depend on the selected user. Interestingly, it is possible to do so.

Proposition 1: Suppose that on channel m and for operator n the regulator selects a single price

$$\lambda_{nk^\star}(m) = \frac{h_{nk^\star}(m)}{2(1 + \bar{\mathbf{p}}_n(m)h_{nk^\star}(m) + \sum_{j\neq n} \bar{\mathbf{p}}_j(m) g_{jk^\star}(m))},$$

where

$$k^\star = \arg \max_{k\in\{1,\ldots,K\}} \frac{h_{nk}(m)}{1 + \sum_{j\neq n} \bar{\mathbf{p}}_j(m) g_{jk}(m)}.$$

denote the index of the strongest user on this channel. Then, the operator n maximizes its utility function on channel m by selecting power $\bar{\mathbf{p}}_n(m)$.

Proof: Clearly if operator n selects user $k^\star$ as defined above, based on the discussion in the previous sub-section it will choose a power $\mathbf{p}_n(m)$. Furthermore if the operator selects another user on this channel then its payoff will only be smaller, since $k^\star$ denotes the user with the largest SINR. Hence the transmitter will maximize its payoff by selecting a power level of $\mathbf{p}_n(m)$. ■

Note that the prices published by the regulator still depend on the knowledge of the channel gains between the operator and the users. Developing mechanisms to learn these channel gains remains an open problem to be addressed in future works.

VII. CONCLUSIONS

We study a joint user-selection and power-allocation game between competitive operators over parallel interference channels. We identify this game as a hybrid game and exploit this structure to develop an algorithm for characterizing the unique NE of the game. A distributed iterative algorithm is also proposed for this game. We develop insights into the user selection problem in high and low SNR regimes via a numerical example. Finally, a pricing mechanism is introduced to drive the NE to any desired operating point in the system.

REFERENCES

[1] "Second report and order and memorandum opinion and order in the matter of unlicensed operation in the TV broadcast bands, additional spectrum for unlicensed devices below 900 MHz and in the 3 GHz band, Federal Communication Commision, Document 08-260," Nov. 2008.

[2] S. Shellhammer, A. K. Sadek, and W. Zhang, "Technical challenges for cognitive radio in the TV white space spectrum," in *Inform. Theory and Applicatoins (ITA)*, 2009.

[3] "IEEE802.22 working group IEEE802.22 Project Homepage, available: http://www.ieee802.org/22/."

[4] "IEEE P802.22/D0.1 Draft Standard for Wireless Regional Area Networks Part 22: Cognitive Wireless RAN Medium Access Control (MAC) and Physical Layer (PHY) specifications: Policies and procedures for operation in the TV Bands."

[5] W. Yu, G. Ginis, and J. Cioffi, "Distributed multiuser power control for digital subscriber lines."

[6] J. S. Pang, G. Scutari, F. Facchinei, and C. Wang, "Distributed power allocation with rate constraints in gaussian parallel interference channels," *IEEE Trans. Inform. Theory*, vol. 54, pp. 3471–89, Aug. 2008.

[7] G. Scutari, D. P. Palomar, and S. Barbarossa, "Asynchronous iterative water-filling for Gaussian frequency-selective interference channels," *IEEE Trans. Inform. Theory*, vol. 54, pp. 2868–78, 2008.

[8] A. Leshem and E. Zehavi, "Game theory and the frequency selective interference channel - a tutorial," March 2009.

[9] Z. Ji and K. J. R. Liu, "Cognitive radios for dynamic spectrum access - dynamic spectrum sharing: A game theoretical overview," *Communications Magazine*, vol. 45, pp. 88–94, May 2007.

[10] F. Wang, M. Krunz, and S. Cui, "Price-based spectrum management in cognitive radio networks," in *Cognitive Radio Oriented Wireless Networks and Communications (CrownCom)*, Aug. 2007, pp. 70–78.

[11] T. M. Cover and J. A. Thomas, *Elements of Information Theory*. John Wiley and Sons, 1991.

[12] T. Alpcan and L. Pavel, "Nash equilibrium design and optimization," in *Intl. Conf. on Game Theory for Networks (GameNets)*, May 2009.