Brief Announcement: Cloud Computing Games: Pricing Services of Large Data Centers

Ashraf Al Daoud[1], Sachin Agarwal[2], and Tansu Alpcan[2]

[1] Department of Electrical and Computer Engineering, Boston University, USA
[2] Deutsche Telekom Laboratories, Technische Universität Berlin, Germany

Organizations opt to reduce costs by contracting their day-to-day computing needs to service providers who offer large-scale data centers and cloud computing services. Like other computing commodities, data centers provide paid services that require careful pricing. Using a Stackelberg game formulation, we present a demand-based pricing model for maximizing revenue of data center providers that serve clients who aim to maximize their utilities.

Problem Formulation: Let d_i denote the demand of client i from the data center quantified by the number of units of processing power per unit time. The service provider (data center) charges the client an amount λ_i per unit demand. So when serving N clients, the revenue of the service provider is $R(\boldsymbol{\lambda}, \mathbf{d}) = \sum_{i=1}^{N} \lambda_i d_i$, where $\boldsymbol{\lambda} = (\lambda_i : i = 1, \ldots, N)$ and $\mathbf{d} = (d_i : i = 1, \ldots, N)$. Clients share the computing resources of the data center. Therefore, the quality of service for one client in terms of processing delay at the data center is affected by the demand of the others. We define the quality-of-service factor of the i^{th} client by $\gamma_i(\mathbf{d}) = L \frac{d_i}{\sum_{k=1, k \neq i}^{N} d_k}$, where $L \geq 1$ is a job decoupling factor which is higher for service providers who provide sufficient computing resources to separate demands of the different clients from affecting each other. If client's utility is logarithmic in the quality of service achieved, then the *net* utility for user i can be given by $U_i(\mathbf{d}, \lambda_i) = \alpha_i log\left(1 + \gamma_i(\mathbf{d})\right) - \lambda_i d_i$, where the constant $\alpha_i > 0$ converts utility to currency.

We consider selfish clients where each is interested in maximizing his net utility. Namely, if client i is charged λ_i per unit demand, and given the demand of the other clients (denoted by d_{-i}), the client's objective is to find d_i^* that solves

$$\textit{Client's problem:} \qquad \max_{d_i \geq 0} \;\; U_i(d_i, d_{-i}, \lambda_i) \;\; \forall i.$$

Given this behavior, the service provider aims to maximize his revenue by imposing optimal prices. The problem can be formulated as a Stackelberg game where the service provider sets prices and consequently the clients update demands (required units of processing power per unit time) to maximize their utilities. Let $\mathbf{d}^*(\boldsymbol{\lambda}) = (d_i^*(\boldsymbol{\lambda}) : i = 1, \ldots, N)$, the service provider's objective is to solve

$$\textit{Service provider's problem:} \qquad \max_{\boldsymbol{\lambda} > \mathbf{0}} \;\; R(\boldsymbol{\lambda}, \mathbf{d}^*(\boldsymbol{\lambda})).$$

Methodology and Main Results: We use a backward induction technique to find Nash Equilibrium (NE) point(s) where neither the service provider nor any of the clients have the incentive to unilaterally deviate. We start with the clients' game and solve for the NE as a function of the price vector $\boldsymbol{\lambda}$. It can be shown that the game of the clients admits a unique NE for any set of prices $\boldsymbol{\lambda} > 0$. In particular, as in [1], index the clients such that if $\frac{\alpha_i}{\lambda_i} < \frac{\alpha_j}{\lambda_j}$ then $i > j$ with the ordering to be picked randomly if $\frac{\alpha_i}{\lambda_i} = \frac{\alpha_j}{\lambda_j}$. Let $M^*(\boldsymbol{\lambda})$ be the largest integer M for which the following condition is satisfied: $\frac{\alpha_M}{\lambda_M} > \frac{1}{L+M-1} \sum_{j=1}^{M} \frac{\alpha_j}{\lambda_j}$. The equilibrium demands of the first $M^*(\boldsymbol{\lambda})$ clients are positive and obtained by $d_i^*(\boldsymbol{\lambda}) = \frac{L}{L-1} \left(\frac{\alpha_i}{\lambda_i} - \frac{1}{(L+M^*(\boldsymbol{\lambda})-1)} \sum_{j=1}^{M^*(\boldsymbol{\lambda})} \frac{\alpha_j}{\lambda_j} \right)$, where $d_i(\boldsymbol{\lambda})^* = 0$ for $i \geq M^*(\boldsymbol{\lambda}) + 1$. We follow on this and solve for the service provider's problem. Namely, let the indexing of the clients be done such that $\sqrt{\alpha_i} < \sqrt{\alpha_j} \Longrightarrow i > j$, with the ordering to be picked arbitrarily if $\sqrt{\alpha_i} = \sqrt{\alpha_j}$. If the following condition is satisfied for all $M \in \{1, \cdots, N\}$

$$\sqrt{\alpha_M} > \frac{1}{L+M-1} \sum_{j=1}^{M} \sqrt{\alpha_j}, \tag{1}$$

then the Stackelberg game admits an infinite number of NE points $(\boldsymbol{\lambda}^*, \mathbf{d}^*)$ where

$$\frac{\lambda_i^*}{\sqrt{\alpha_i}} = \frac{\lambda_j^*}{\sqrt{\alpha_j}}, \quad \forall i, j = 1, \cdots, N. \tag{2}$$

The corresponding demand levels are non-zero and given by

$$d_i^* = \frac{L}{(L-1)} \frac{1}{\lambda_i^*} \left(\alpha_i - \frac{\sqrt{\alpha_i}}{(L+N-1)} \sum_{j=1}^{N} \sqrt{\alpha_j} \right).$$

We give a proof of this result in an analogous setup in [2]. Formula (2) devises prices that are optimal over the set of prices that result in all the clients to have non-zero demand levels. It has a proportional structure that suggests charging more the clients that are more willing to pay for their utilities, i.e. higher α's. Here, if L is large enough, then condition (1) is satisfied for all $M \in \{1, \cdots, N\}$. Intuitively, the higher the decoupling factor L is, the better the quality of service for the client and the lesser the external effect due to the other clients' demands. Therefore, there is an incentive for the clients to have non-zero demand levels. We can also show that if condition (1) is satisfied, it is suboptimal for the service provider to drop any of the clients by imposing a sufficiently high price. In other words, under that condition, the suggested prices in (2) are optimal over the set $\boldsymbol{\lambda} > 0$.

References

1. Alpcan, T., Basar, T., Srikant, R., Altman, E.: "CDMA Uplink Power Control as a Noncooperative Game," *IEEE Trans. on Wireless Net.*, vol. 8, pp. 659–670, (2002)
2. Al Daoud, A., Alpcan, T., Agarwal, S., Alanyali, M.: "A Stackelberg Game for Pricing Uplink Power in Wide-Band Cognitive Radio Networks," *In Proc. of 47th IEEE Conference on Descision and Control*, pp. 1422–1427, (2008)